Combinable products, price discrimination, and collusion*

Hendrik Döpper [a], Alexander Rasch [a,b]

[a] *Düsseldorf Institute for Competition Economics, Heinrich-Heine-University Düsseldorf, Universitätsstraße 1, 40225 Düsseldorf*
[b] *Leibniz Centre for European Economic Research (ZEW)*

August 29, 2023

Abstract

We analyze the effect of different pricing schemes on horizontally differentiated firms' ability to sustain collusion when customers have the possibility to combine (or mix) products to achieve a better match of their preferences. To this end, we compare two-part tariffs with linear prices and quantity-independent fixed fees. We find that a ban of either price component of the two-part tariff makes it more difficult to sustain collusion at profit-maximizing prices. Moreover, linear pricing—as the most beneficial pricing schedule for customers in absence of collusion—harms customers most in presence of collusion.

Keywords: Collusion; Combinable products; Mixing; Price discrimination; Two-part tariff.
JEL: D43; L13; L41.

1. Introduction

The present paper contributes to the ongoing debate in competition and customer protection policy that centers around the competitive or anti-competitive effects of different pricing schemes. In our analysis, we focus on the aspect of combining products (or mixing) and further explore the framework applied by Anderson and Neven (1989) and Hoernig and Valletti (2007, 2011). Under the possibility of mixing, customers can demand products from different firms, and in doing so, they create their own products that better fit their needs and preferences. The idea is that, from the customers' perspective, each product has advantages and disadvantages, and by combining different products, customers can enjoy the benefits of different products while offsetting the disadvantages of each product. In this sense, the characteristics of the new individualized "product" are a combination of the characteristics of the individual products. The possibility to combine products is a widespread phenomenon in many important industries (for example, banking and insurance; see our discussion below).

*We would like to thank Bernhard Kasberger, Nicholas Shunda, Achim Wambach, David Zeimentz, and participants at the CRESSE 2019, the EARIE 2019, the Competition and Innovation Summer School 2019, and the 4th DICE Winter School 2019 for very helpful comments and discussions. We also thank the Co-Editor, José Moraga, and two anonymous referees. Computational infrastructure and support were provided by the Centre for Information and Media Technology at Heinrich Heine University Düsseldorf. Funding by the Deutsche Forschungsgemeinschaft (DFG) (project 235577387/GRK1974) is gratefully acknowledged.

Email addresses: doepper@dice.hhu.de (Hendrik Döpper), rasch@dice.hhu.de (Alexander Rasch)

The scope of mixing (and, hence, the extent of product match) is crucial for welfare and policy considerations. In our framework, each customer buys one unit. The customers realize a basic utility from consumption, and prices are mere transfers of customer utility to the firms. The only source of disutility that may lower total welfare comes from a mismatch of customers' preferences that reduces customer surplus. Previous contributions analyze how different pricing schemes affect the scope of mixing and find that simple pricing schemes can be beneficial for customers. The best outcome is observed with linear prices. In this case, all customers optimally combine products, so that welfare is maximized. By contrast, the worst outcome is achieved with quantity-independent fixed fees that lead to a non-mixing result, that is, all customers buy exclusively from one firm. As a consequence, customers do not buy their preferred products and, thus, suffer from a disutility, which in turn leads to a welfare loss. With two-part tariffs and nonlinear pricing in general, some mixing occurs, so that welfare ranks second after the case of linear prices. Therefore, regulations of the pricing regimes, such as a ban of the fixed price component of a two-part tariff, can increase the number of customers who combine their products and, hence, increase customer surplus and total welfare.

Whereas the aforementioned contributions focus on a static environment, we extend the framework to a dynamic model to investigate firms' incentives to collude in an infinitely repeated game. By applying grim-trigger strategies, we derive the critical discount factors to compare the impact of linear prices, fixed fees, and two-part tariffs on firms' ability to collude. We show that firms' access to multi-part pricing schemes can reduce the scope for collusion in dynamic environments. More precisely, our main result is that firms' ability to use two-part tariffs can reduce collusion. In this sense, the effect of restrictions on pricing schemes are less clear-cut, and regulations that are explicitly imposed to benefit customers can backfire.

We investigate the robustness of this result with respect to two important assumptions. First, we demonstrate that the consequence of banning price discrimination does not only hold for two-part tariffs, but also extends to nonlinear pricing in general. Second, we allow firms to collude on prices below the profit-maximizing collusive prices (partial collusion). We find that firms are most likely to gain the largest profits when they partially collude on linear prices instead of fixed fees or two-part tariffs. By contrast, partial collusion on fixed prices does not yield any advantage. This finding complements the result of our baseline analysis in the sense that linear pricing schedules are most beneficial for customers in the absence of collusion, but they harm customers most in the presence of (partial) collusion.

Our analysis sheds light on the potential for anti-competitive practices in some of the most essential sectors of the economy. Prime examples for markets to which our results can be applied are the banking and the insurance sector and related services. These markets are indeed characterized by the four most important ingredients of our model: (i) mixing, (ii) two-part tariffs (or, more generally, nonlinear pricing), (iii) regulation of/ban on certain tariff components, and (iv) collusion. Having a closer look at the market for banking services shows that in the United States, for example, people have three credit cards on average.[1] Similarly, half of Americans use more than one bank, and it is recommended to have four accounts (at different banks).[2] Moreover, credit card and banking services usually come at various fees. As an example, credit card holders and bank account owners pay an annual fixed fee and linear fees for additional services (for instance, credit card usage, cash withdrawal).[3] Furthermore, the industry has seen various efforts by regulators to cap or ban certain fees.[4] Regulators have particularly focused on overdraft fees that come in different forms (fixed, linear, nonlinear).[5] In the UK, the Financial Conduct Authority (FCA) banned fixed fees for borrowing through an overdraft in 2019. Banks and building societies are now required to price overdrafts by a simple annual interest rate. A similar approach by the Consumer Financial Protection Bureau (CFPB) is currently under way in the United States, where the CFPB intends to intervene in overdraft practices. In this context, large banks such as Capital One and Bank of America decided to end or greatly reduce the use of these fees, and it has been argued that they had done so in anticipation of regulatory intervention.[6] With regard to regulating fees for credit cards, the Credit Card Accountability Responsibility and Disclosure Act of 2009 (CARD Act) stipulates that a credit card late payment fee must not surpass $30 for a first late payment.[7] On a related note, the prohibition of surcharges for credit cards above a certain percentage level in some US states and other jurisdictions (for example, Australia) can be viewed as an indirect form of regulating the linear pricing component. Because merchants' adoption decision is crucially affected by the credit card firms' interchange fee, credit card firms are restricted in their pricing strategy. In the European Union, there is a direct link between interchange regulation and the no-surcharge rule: "In all cases where the card charges imposed on merchants will be capped, in accordance with the complimentary multilateral interchange fees (MIF) Regulation (...), merchants will no longer be

[1] See https://www.forbes.com/advisor/credit-cards/credit-card-statistics/.

[2] See https://www.gobankingrates.com/banking/banks/how-many-bank-accounts-americans-have/, https://www.businessinsider.com/personal-finance/how-many-bank-accounts-should-i-have-tiffany-aliche, and https://www.gobankingrates.com/banking/checking-account/how-many-bank-accounts-can-have/.

[3] Note that these fees may be negative, for example, because firms offer frequent-flyer points and insurance coverage as extra benefits.

[4] On a general note, the Biden-Harris Administration has recently launched a campaign against so-called junk fees and related pricing practices. One of the sectors targeted is banking (see https://www.whitehouse.gov/briefing-room/blog/2022/10/26/the-presidents-initiative-on-junk-fees-and-related-pricing-practices/). Other examples include airline pricing, event ticketing, and hotel booking.

[5] See https://www.moneysavingexpert.com/news/2018/12/fixed-daily-and-monthly-overdraft-charges-to-be-banned/.

[6] See https://www.theregreview.org/2022/01/22/saturday-seminar-united-states-over-overdraft-fees/.

[7] See https://www.bankrate.com/finance/credit-cards/late-fee-on-a-credit-card-late-fee/.

allowed to surcharge customers for using their payment card."[8] Turning to the fourth characteristic, the banking sector has seen quite a few cases of collusion. In a recent case in the United States, eight banks are being investigated because they are suspected of artificially blowing up interest rates that state and local governments must pay on a popular tax-exempt municipal bond.[9] Another case of anti-competitive behavior concerns initial public offerings. In 2017, the OECD concluded that banks' high fees were "akin to tacit collusion."[10]

With regard to the application of our set-up to the insurance sector, we point out that in the market for insurance services, mixing is also a widespread phenomenon. For example, a study for the German insurance market found that on average people have contracts with 2.7 insurance companies (Bain & Company, 2012). Such insurance contracts often include different price components; for example, a fixed annual fee and a deductible (either in fixed or in percentage terms). Furthermore, regulation is an important aspect in this sector. For example, in Germany, insurers and brokers cannot pass on commissions to customers.[11] Bans on upfront commissions are also being discussed in other parts of the world.[12] The Central Bank of Ireland recently introduced a ban on so-called "price walking" for private car and home insurance customers. Under this policy, discounts to attract new customers have to be made available also to those customers who remain with the same insurance provider (no loyalty penalty).[13] Last, price-fixing practices are an issue in the industry as exemplified by the discussions at an OECD policy roundtable (OECD, 1998).

The remainder of this paper is organized as follows. In the next section, we discuss the related literature. We describe the model and previous findings in Section 3. Section 4 analyzes collusion when firms can set profit-maximizing prices (full collusion). We investigate the robustness of our findings in Section 5. In doing so, we start with a general comment suggesting that the result with regard to the comparison of the stability of collusion with linear prices and two-part tariff can be expected to be robust to changes in many assumptions (Section 5.1). We discuss so-called fully nonlinear tariffs in Section 5.2 and partial collusion

[8] https://ec.europa.eu/commission/presscorner/detail/de/MEMO_13_719. In Europe, the interchange is capped at 0.3% for credit cards, such that merchants are estimated to be indifferent between accepting payment by card or in cash. In most of those US states where surcharging is legal, the surcharge is capped at 4% of the transaction total. In any case, it must not be used by merchants to make an extra profit.

[9] See https://www.reuters.com/business/finance/judge-narrows-san-diego-baltimore-bond-collusion-cases-against-big-banks-2022-06-28/.

[10] See https://www.theglobeandmail.com/report-on-business/streetwise/the-tacit-collusion-of-big-bank-fee-setting/article35156863/. Chen and Ritter (2000) provide evidence for this observation. Hansen (2001) argues that profits are not abnormal in the industry.

[11] See https://www.lexology.com/commentary/insurance/germany/arnecke-sibeth-dabelstein/prohibition-on-passing-on-commission-in-reinsurance-context-exemption-uncertainty.

[12] See, for example, https://www.investmentexecutive.com/news/from-the-regulators/insurance-regulators-start-consultation-on-banning-upfront-commissions/.

[13] See https://www.centralbank.ie/news-media/press-releases/press-release-end-the-loyalty-penalty-for-private-car-and-home-insurance-21-July-2021. As such, the policy hinders insurance firms to price-discriminate.

in Section 5.3. We summarize our findings and discuss possible limitations of our model in Section 6. All proofs are relegated to Appendix A.

2. Related literature

We first contribute to the literature on combinable products. Customers' possibility to mix different products was first analyzed by Anderson and Neven (1989) for the case with linear prices and was later adopted by, among others, Gal-Or and Dukes (2003) and Gabszewicz et al. (2004) to analyze media markets. Hoernig and Valletti (2007, 2011) investigate the impact of different pricing schemes by analyzing two-part tariffs and nonlinear pricing in general. As noted in the introduction, the contributions of Anderson and Neven (1989) and Hoernig and Valletti (2007, 2011) show that the scope of mixing crucially depends on the pricing policy. Whereas customers buy from one firm exclusively and, hence, do not mix at all if firms charge fixed prices, customers optimally mix in the sense that they get a perfect match of their preferences if firms charge linear prices. With two-part tariffs and nonlinear pricing in general, some mixing occurs: Only those customers whose preferences are met worst combine products to achieve a better fit. In a static environment, Hoernig and Valletti (2007) stress that the main and robust result is that firm profits are higher as the number of pricing instruments increases.[14] Because mixing benefits customers under competition, the contributions highlight that regulations of the pricing regimes can benefit customers. We add to this strand of literature by analyzing the impact that such regulations can cause in dynamic environments.

We contribute to the literature on the interplay between price discrimination and collusion.[15] Although price discrimination has been an important topic in the antitrust community, the literature on the effects of price discrimination on collusion is rather limited. Two-part tariffs are a classic tool to price-discriminate between customers. Thus, a ban of one of the two price components corresponds to a ban of price discrimination. Gössl and Rasch (2020) are the closest to us in that they use a Hotelling (1929) framework with linear transport costs and elastic demand to study how a ban of either the linear or fixed price component of a two-part tariff affects the ability of firms to sustain collusion. The underlying mechanisms driving the models are fundamentally different. In the traditional Hotelling (1929) framework, firms compete for the indifferent customer and the location of that customer alone determines from which firms customers buy. By contrast, firms face a more complex demand structure in our model in which each customer can decide

[14] This result is different from the related literature on mixed bundling, which uses mix-and-match models and shows that profits are lower with more instruments, particularly in the case in which firms practice mixed bundling compared to the situation in which products are sold separately (see Matutes and Regibeau, 1992).

[15] Although we focus on models with horizontal product differentiation, a related strand of literature has emerged in the context of vertical product differentiation. The process of customers self-selecting into their preferred quality levels is a form of second-degree price discrimination. In such a set-up, Häckner (1994), Symeonidis (1999), and Bos and Marini (2019) analyze the sustainability of collusive agreements. Bos et al. (2020) further explore the role of cartel formation.

whether the customer wants to buy from one firm exclusively or combine products from both firms. This possibility to mix can result in two indifferent customers and a share of customers who granularly adjust their demand at both firms when prices change. The different mechanisms at work correspond to different industries. Given the underlying difference in both set-ups, it is not surprising that the results diverge too: Among other things, Gössl and Rasch (2020) find that a ban on linear prices facilitates collusion, whereas a ban on fixed fees hampers collusion. This is in stark contrast to our finding that both types of ban indeed facilitate collusion.

Both Gössl and Rasch (2020) and our work consider second-degree price discrimination. Liu and Serfes (2007) use a model à la Hotelling (1929) with linear transport costs to analyze the impact of customer-specific information (that is, third-degree price discrimination) on tacit collusion. In their framework, information gives firms the ability to distinguish between different subintervals (market segments) of the linear city. A higher quality of information results in smaller subintervals and, hence, in more market segments. Firms can perfectly identify their customers by their segment and charge prices based on this information. The authors find that collusion becomes less likely as the quality of information (that is, the number of market segments) increases.[16] Liu and Serfes (2007) also show that this result is not clear ex ante because of two opposing channels. The more information firms have, the better they can target their customers. On the one hand, this leads to higher collusive profits and harsher punishment; but on the other hand, deviation profits increase as well. Our results are similar in the sense that multi-part pricing schemes allow firms to better target their customers and make collusion at profit-maximizing prices more difficult. Note, however, that we analyze a situation with second-degree price discrimination because firms cannot distinguish between customers who self-select into their preferred mixing choice.

Whereas Liu and Serfes (2007) assume that firms have perfect information in the sense that they can identify customers of each market segment with certainty, Colombo and Pignataro (2022) and Peiseler et al. (2022) investigate the role of imperfect information about customers' locations on firms' ability to collude. In both set-ups, firms try to distinguish between their loyal customers located in their own turf and disloyal customers located in their rival's turf. In Colombo and Pignataro (2022), the signal quality determines which share of loyal customers a firm can identify. It is, however, not possible to distinguish unidentified loyal customers from disloyal customers. This is different in Peiseler et al. (2022), where the signal quality

[16] Colombo (2010) analyzes the impact of product differentiation on the stability of collusion in the limiting case in which the number of market segments approaches infinity (delivered pricing/perfect price discrimination). He picks up the prevalent finding in the literature that the relationship between the degree of product differentiation and firms' ability to collude is positive in the context of the Hotelling (1929) framework (see, for example, Chang, 1991, 1992 and Häckner, 1995; Ross, 1992 is a notable exception). He finds no relationship between product differentiation and the likelihood of collusion if firms collude on either customer-specific prices or on the decision whether to apply price discrimination (but not on the price level). In addition, he finds a negative relationship when firms collude on a uniform price. Further contributions that examine the effects of delivered prices on collusion include Jorge and Pires (2008) and Miklós-Thal (2008).

determines the ability of firms to identify the brand loyalty of an arbitrary customer. This difference gives rise to different results: Whereas Peiseler et al. (2022) find that a ban of price discrimination hampers collusion for a sufficiently low signal quality, Colombo and Pignataro (2022) find that a ban of price discrimination hampers collusion for large degrees of product differentiation and a sufficiently large signal quality.

Finally, Helfrich and Herweg (2016) also analyze the effect of third-degree price discrimination on collusion in a linear city. The authors find that a ban of price discrimination raises the ability of firms to sustain collusion. The findings of Helfrich and Herweg (2016) are similar to ours in the sense that in our paper, a ban of price discrimination also facilitates collusion. However, in contrast to Helfrich and Herweg (2016), we analyze second-degree instead of third-degree price discrimination and consider various extensions that entail important consequences. For instance, we allow firms to collude on prices below the profit-maximizing prices (partial collusion).[17]

3. Model and previous findings

We adopt the models of Anderson and Neven (1989) and Hoernig and Valletti (2007) and consider two horizontally differentiated, symmetric firms that are located at the end points of a linear city of unit length (Hotelling, 1929). Fixed and marginal costs are normalized to zero. Firms discount future profits by the common discount factor δ per period. We analyze three different pricing regimes. Firms charge either linear prices $p_{i,L}$ per unit purchased, fixed fees $f_{i,F}$ that are independent of actual usage, or two-part tariffs $(p_{i,T}, f_{i,T})$ that include both a linear and a fixed price. The different scenarios are denoted by the subscripts L, F, and T.

Customers of mass one are uniformly distributed along the line. Each customer has a total demand of zero or one. Customers have a basic valuation of v for each product and incur transport costs. Transport costs reflect the fact that customers' preferences are not fully matched by the firms' products, that is, a customer located at x incurs quadratic transport costs of τx^2 or $\tau(1-x)^2$ when buying only from firm 1 or firm 2.

So far, the application of the transport costs follows the standard logic of the classic framework in Hotelling (1929). The modification of Anderson and Neven (1989) is to allow customers to save on these costs by splitting their demand across the two firms to purchase an optimal individual mix of both products. Let λ (with $0 \leq \lambda \leq 1$) denote the share of overall demand that a customer buys from firm 1; the remaining share of $1-\lambda$ is bought from firm 2. Then, mixing leads to transport costs of

$$\tau(\lambda \cdot 0 + (1-\lambda) \cdot 1 - x)^2 = \tau(1-\lambda-x)^2.$$

[17] Horstmann and Krämer (2013) analyze the impact of third-degree price discrimination on collusive outcomes in an experimental setting. In contrast to theoretical predictions, the authors find that third-degree price discrimination leads to significantly higher prices and profits compared to uniform pricing.

The first part in brackets $\lambda \cdot 0 + (1-\lambda) \cdot 1$ is the location of the new product, which results from combining the products of both firms. It is a weighted combination of the locations of the individual products and, thus, depends on the decision of the customer about λ. The new location is then evaluated against the location of the customer, x, and the difference in the locations describes the distance that is used to calculate the transport costs.

Another way to think about the transport costs is to rewrite the above expression in the following way

$$\tau \left(\lambda \cdot (0 - x) + (1 - \lambda) \cdot (1 - x) \right)^2 .$$

Here, we have a weighted combination of the distances between the customer's location and both firm locations. Note that the distance to the product of firm 1 is (weakly) negative and the distance to the product of firm 2 is (weakly) positive. The important difference between the standard Hotelling (1929) framework and the model of Anderson and Neven (1989) is that we assign an interpretation to the sign of a distance. The idea is that each product comes with advantages and disadvantages (from the viewpoint of each customer). For instance, we can think of two credit cards that differ in their usage benefits for different customers. Both credit cards can be used both at home and abroad, but they differ in their acceptance rate: One credit card can be used more frequently in one country (or region), whereas the other card is more prominently accepted in another. In this case, those customers who mostly shop in their home country and only seldom go abroad can be expected to carry the credit card that is widely accepted at home. By contrast, customers who often go on business trips or on holiday abroad may want to contemplate whether to carry both cards to gain more flexibility. The model of Anderson and Neven (1989) captures this by allowing negative and positive distances to offset each other. Because both products are located at the extremes, λ can be chosen to achieve an arbitrary location of the new product on the line $[0, 1]$. If the customer wants to fully save on the transport costs, the customer has to select λ such that the location of the new product equals exactly his/her own location.

Even though (complete) savings on transport costs are possible, transport costs usually affect prices because they are a measure of firms' market power. Typically, the focus in models with horizontal product differentiation is on situations in which the market is covered, that is, the basic valuation v must be relatively large compared to the transport costs, so that the utility in equilibrium is not negative and customers disregard the outside option not to buy. We also focus on this case and make the following assumption:

Assumption 1. Transport costs are not too high relative to the basic valuation from buying, that is, $0 < \tau \leq 4v/5$.

Customer face three different potential utilities, depending on where they buy. For the case of two-part tariffs, the following utility functions U refer to the cases in which the customer located at x buys exclusively from firm 1, buys exclusively from firm 2, or combines the products of both firms (dropping subscripts for the different pricing scenarios for now):

$$U_1(x) = v - f_1 - p_1 - \tau x^2,$$
$$U_2(x) = v - f_2 - p_2 - \tau(1-x)^2,$$
$$U_m(x) = v - f_1 - f_2 - \lambda p_1 - (1-\lambda)p_2 - \tau\left(1-\lambda-x\right)^2.$$

A mixing customer will optimally choose share λ to maximize utility depending on the location, that is,

$$\frac{\partial U_m}{\partial \lambda} = 0 \quad \Leftrightarrow \quad \lambda\left(x\right) = 1 - x - \frac{p_1 - p_2}{2\tau}.^{18} \tag{1}$$

Given the decision about the optimal share, we derive the customer who is indifferent between buying exclusively from firm 1 and mixing. Denote this customer's location by $\underline{x}$. Similarly, denote the location of the customer who is indifferent between mixing and buying exclusively from firm 2 by $\overline{x}$. Assume that $0 \leq \underline{x} \leq \overline{x} \leq 1$ holds. Then, the locations of the indifferent customers are given by

$$U_1\left(\underline{x}\right) = U_m\left(\underline{x}\right) \quad \Leftrightarrow \quad \underline{x} = \sqrt{\frac{f_2}{\tau}} - \frac{p_1 - p_2}{2\tau},$$
$$U_m\left(\overline{x}\right) = U_2\left(\overline{x}\right) \quad \Leftrightarrow \quad \overline{x} = 1 - \sqrt{\frac{f_1}{\tau}} - \frac{p_1 - p_2}{2\tau}.$$

For $0 \leq \underline{x} \leq \overline{x} \leq 1$, the profit function of each firm consists of three parts:

$$\pi_1\left(f_1, p_1; f_2, p_2\right) = f_1 \overline{x} + p_1 \underline{x} + p_1 \int_{\underline{x}}^{\overline{x}} \lambda\left(x\right) dx,$$

$$\pi_2\left(f_1, p_1; f_2, p_2\right) = f_2\left(1 - \underline{x}\right) + p_2\left(1 - \overline{x}\right) + p_2 \int_{\underline{x}}^{\overline{x}} \left(1 - \lambda\left(x\right)\right) dx.$$

The first part consists of the fixed fee that is paid by both loyal and mixing customers. The second and third parts quantify the linear payments. Whereas loyal customers buy exclusively from one firm and, hence, pay the full linear price (second part), mixing customers buy their optimal shares that depend on their locations (third part).

If $\underline{x} > \overline{x}$, customers never mix. In this case, we are back in the classic Hotelling (1929) game with quadratic transport costs as analyzed by d'Aspremont et al. (1979).

Further note that the cases of linear prices and fixed fees are special cases of two-part tariffs. All formulas for these cases follow immediately from setting the respective price component equal to zero.

Before we turn to analyzing the cases of collusion and deviation, we briefly recap the results in the competitive scenarios (denoted by an asterisk) that are derived in Anderson and Neven (1989) and Hoernig and Valletti (2007) in the static one-shot game:

[18] Note that the second-order condition is satisfied, that is, $\partial^2 U_m / \partial \lambda^2 = -2\tau < 0$.

Recap 1. *Competitive prices are given by*

$$p_L^* = f_F^* = \tau \qquad \textit{(linear and fixed prices)},$$

$$(f_T^*, p_T^*) = \left(\frac{\left(7 - 3\sqrt{5}\right)\tau}{2}, \frac{\left(3\sqrt{5} - 5\right)\tau}{2} \right) \qquad \textit{(two-part tariffs)}.$$

Competitive profits amount to

$$\pi_L^* = \pi_F^* = \frac{\tau}{2} \qquad \textit{(linear and fixed prices)},$$

$$\pi_T^* = \frac{\left(13\sqrt{5} - 27\right)\tau}{4} \qquad \textit{(two-part tariffs)}.$$

Customer surplus and welfare is given by

$$CS_L^* = v - \tau \qquad \textit{and} \quad W_L^* = v \qquad \textit{(linear prices)},$$

$$CS_F^* = v - \frac{13\tau}{12} \qquad \textit{and} \quad W_F^* = v - \frac{\tau}{12} \qquad \textit{(fixed prices)},$$

$$CS_T^* = v - \frac{\tau(23\sqrt{5} - 45)}{6} \qquad \textit{and} \quad W_T^* = v - \frac{\tau\left(3 - \sqrt{5}\right)^3}{12} \qquad \textit{(two-part tariffs)}.$$

Although linear and fixed prices (and profits) are the same, they lead to remarkably different market outcomes. In the case of linear prices, all customers buy their optimal mix, such that transport costs are zero. As a result, welfare is maximized. By contrast, customers do not mix in the case of fixed fees, and, hence, the outcome is the same as in the classic game analyzed by d'Aspremont et al. (1979). The total welfare loss due to transport costs is $\tau/12$, and customers are worse off.

Two-part tariffs enable firms to segment customers. By setting a strictly positive fixed fee, firms extract additional surplus from their loyal customers. In turn, they are willing to lose extremely disloyal customers who are located close to their competitor and would only buy a small share anyway. In contrast to pure fixed pricing, the fixed-price component is lower, so that it is beneficial for customers around the center of the linear city to mix. These customers face higher transport costs compared to the loyal customers and, hence, are willing to pay the fixed fee twice to save on these costs.

Because some customers mix and others do not, the welfare loss through transport costs is lower than in the case of fixed fees only, but larger than in the case of linear pricing. Although the additional price component in the case of a two-part tariff allows firms to extract additional surplus from loyal customers, the decline in the overall transport costs is so large compared to the case of fixed pricing, such that customers benefit overall, that is, customer surplus is larger under two-part tariffs than under fixed fees only. However, customer surplus is largest under linear prices because customers face zero transport costs and firms extract less surplus.

4. Collusion at maximum prices

In our main analysis, we focus on three scenarios that relate to the situations discussed in the Introduction: (i) Firms can only set linear prices due to a regulation that does not allow the use (or setting) of fixed fees; (ii) firms can only choose fixed fees because linear prices are banned (or regulated); and (iii) firms are unrestricted in their price-setting and may choose linear and fixed prices (no ban or regulation).

Collusive strategy

Throughout our analysis, we adopt the critical discount factor as a measure for the likelihood of (full) collusion. To this end, we focus on the standard grim-trigger strategies defined by Friedman (1971).[19] Denote the profits in the cases of collusion and deviation by π^c and π^d. Then, collusion is profitable as long as the discounted profits from collusion are higher than those from deviation and the ensuing punishment phase, that is,

$$\sum_{t=0}^{\infty} \delta^t \pi^c \geq \pi^d + \sum_{t=1}^{\infty} \delta^t \pi^*.$$

Hence, collusion can be sustained for any discount factor larger than the critical discount factor defined as

$$\overline{\delta} := \frac{\pi^d - \pi^c}{\pi^d - \pi^*}. \tag{2}$$

As a consequence, collusion is facilitated when the critical discount factor decreases because firms can sustain collusion for a larger range of discount factors.

Because we already discussed the competitive profits, we directly turn to the remaining cases of collusion and deviation.

Collusive outcomes

The following lemma summarizes the collusive outcomes:

[19] Apart from grim-trigger strategies, where firms return to Nash pricing after collusion is detected, other punishment strategies are possible. Most notable are optimal punishment strategies following the seminal work in Abreu (1986, 1988) and Abreu et al. (1986). In the context of the Hotelling (1929) framework, there is tentative evidence that optimal punishment strategies lead to similar results compared to grim-trigger strategies. For example, Häckner (1996) uses a standard set-up with quadratic transport costs and symmetric firms and shows that the impact of product differentiation on collusive prices is qualitatively similar with optimal punishment compared to the results achieved by Chang (1991) with grim-trigger strategies. Furthermore, in the context of price discrimination, Liu and Serfes (2007) report that their main result that is derived in a Hotelling (1929) set-up with linear transport costs is also robust when they move from grim-trigger to stick-and-carrot punishments. Because optimal punishment strategies come at the expense of less tractable models, we stick to grim-trigger strategies.

Lemma 1. *Collusive prices are given by*

$$p_L^c = v \qquad \textit{(linear prices)},$$
$$f_F^c = v - \frac{\tau}{4} \qquad \textit{(fixed prices)},$$
$$(p_T^c, f_T^c) = (v, 0) \qquad \textit{(two-part tariffs)}.$$

Collusive profits amount to

$$\pi_L^c = \pi_T^c = \frac{v}{2} \qquad \textit{(linear prices and two-part tariffs)},$$
$$\pi_F^c = \frac{v}{2} - \frac{\tau}{8} \qquad \textit{(fixed prices)}.$$

Customer surplus and welfare is given by

$$CS_L^* = 0 \quad \textit{and} \quad W_L^* = v \qquad \textit{(linear prices)},$$
$$CS_F^* = \frac{\tau}{6} \quad \textit{and} \quad W_F^* = v - \frac{\tau}{12} \qquad \textit{(fixed prices)},$$
$$CS_T^* = 0 \quad \textit{and} \quad W_T^* = v \qquad \textit{(two-part tariffs)}.$$

With linear prices, firms charge prices that are equal to the basic valuation. The reason for this behavior can be explained by two effects. First, by setting equal prices, all customers buy their optimal mix and, hence, do not incur transport costs. As a consequence, firms maximize customers' utility. Second, by setting the price level to the basic valuation, firms fully extract the maximized utility, such that producer surplus equals the maximized welfare and customer surplus is zero.

Because firms gain the highest possible profits with linear prices, firms cannot take advantage of the additional fixed fee in the case of two-part tariffs. A strictly positive fixed fee would lead to a share of customers who do not mix and, hence, suffer from a loss in utility due to strictly positive transport costs. As a consequence, firms set the fixed component equal to zero and charge the linear price equal to the basic valuation. Again, welfare is maximized and equals producer surplus; customer surplus is zero.

Finally, customers do not mix with fixed-fee pricing, and firms are in the same situation as in the classic set-up analyzed by Chang (1991). They set the optimal fixed fees, such that the indifferent customer at the center is indifferent between buying and not buying. As a result, all customers incur strictly positive transport costs, which leads to a welfare loss of $\tau/12$. However, in contrast to the other two pricing environments, customer surplus is strictly positive.

Deviation

Based on the collusive outcomes, we determine optimal prices and profits of a deviating firm:

Lemma 2. *Define* $A := \sqrt{v^2 - 4v\tau + 28\tau^2}$. *Optimal deviation prices are given by*

$$p_L^d = \frac{2v - 4\tau + A}{3} \qquad \text{(linear prices)},$$

$$f_F^d = \begin{cases} v - \frac{5\tau}{4} & \text{if } 0 < \tau \leq \frac{4v}{13} \\ \frac{v}{2} + \frac{3\tau}{8} & \text{if } \frac{4v}{13} < \tau \leq \frac{4v}{5} \end{cases} \qquad \text{(fixed prices)},$$

$$\left(f_T^d, p_T^d\right) = (\tau, v - 2\tau) \quad \text{if } 0 < \tau \leq \frac{v}{4} \qquad \text{(two-part tariffs)}.$$

For $\tau > v/4$, *we use*

$$\left(f_T^d, p_T^d\right) = \left(\frac{(v-\tau)^2}{9\tau}, \frac{v + 2\tau}{3}\right)$$

to calculate a bound on the profit for two-part tariffs. Then, optimal deviation profits amount to

$$\pi_L^d = \frac{(-2v + 4\tau - A)(v^2 - vA - 4v\tau + 2\tau A - 20\tau^2)}{108\tau^2} \qquad \text{(linear prices)},$$

$$\pi_F^d = \begin{cases} v - \frac{5\tau}{4} & \text{if } 0 < \tau \leq \frac{4v}{13} \\ \frac{(4v+3\tau)^2}{128\tau} & \text{if } \frac{4v}{13} < \tau \leq \frac{4v}{5} \end{cases} \qquad \text{(fixed prices)},$$

$$\pi_T^d \begin{cases} = v - \tau & \text{if } 0 < \tau \leq \frac{v}{4} \\ \geq \frac{-v^3 + 12v^2\tau + 6v\tau^2 + 10\tau^3}{54\tau^2} & \text{if } \frac{v}{4} < \tau \leq \frac{4v}{5} \end{cases} \qquad \text{(two-part tariffs)}.$$

Consider the case of linear prices first. The deviating firm sets its price, such that it serves a loyal customer base exclusively and sells shares of its product to disloyal customers. Thus, it never monopolizes the market, but leaves its competitor always with a strictly positive market share.

In contrast to linear prices, customers do not mix neither under competition nor under collusion in the case of fixed fees. This enables a deviating firm to monopolize the whole market if product differentiation is sufficiently low (that is, $0 < \tau \leq 4v/13$). The monopolization requires that the firm compensates the farthest customer for the transport costs. With an increasing degree of product differentiation (that is, $\tau > 4v/13$), this compensation becomes unattractive, so that the deviating firm leaves its competitor with a strictly positive market share.

The case of two-part tariffs poses some challenges. Although the mathematical problem is well-defined and it is possible to write down the optimization problem of the deviating firm, it is difficult to derive closed-form solutions for all cases because of the highly nonlinear nature of some equations. In principle, the deviating firm can set its prices in three different ways. First, it can set its prices to monopolize the market. Second, it can cover the entire market, but leave its rival with a strictly positive market share, that is, some customers close to the rival's location buy from both firms. The third option is to set prices, such that it does not cover the entire market. In this case, some customers who are located close to the rival's location choose the outside option and do not buy. The reason is that the rival charges a linear price of v which equals the basic utility. This means that after paying the price, a customer would have zero utility. All customers except the marginal customer at $x = 1$ would also have to pay transport costs on top, resulting in a negative utility. Therefore, these customers refrain from buying from any firm. In addition, there would be two other groups of customers, those who are located near the deviator's location and buy exclusively from that firm and those who are located towards the center and buy from both firms.

It is possible to derive closed-form solutions for the prices and profits in the first two cases. These two cases allow us to derive a lower bound for the profit. In addition, we will discuss in Section 5.2 that the profit in the case of fully nonlinear tariffs can serve as an upper bound. This allows us to provide an even more precise characterization for the case $\tau \leq v/4$ because the lower bound equals the upper bound. This allows us to conclude that in this case, the deviating firm monopolizes the market and serves all customers.

Table 1 provides a summary of our results so far to help to understand the mechanism behind Proposition 1 below.

Critical discount factors

The following lemma summarizes the critical discount factors that result from inserting the outcomes for the cases of collusion, deviation, and competition into Expression (2) for the critical discount factor. For the case of two-part tariffs, we use the lower bound on the deviation profit to obtain a lower bound for the corresponding critical discount factor. This makes sense because a larger deviation profit ceteris paribus renders deviation more attractive and thus leads to an increase in the critical discount factor.

Lemma 3. *Define $B := v^3 - v^2A - 6v^2\tau + 4v\tau A - 28\tau^2 A$. The critical discount factors are given by*

$$\overline{\delta}_L = \frac{B - 6v\tau^2 + 136\tau^3}{B - 60v\tau^2 + 190\tau^3} \qquad (\textit{linear prices}),$$

$$\overline{\delta}_F = \begin{cases} \frac{4v-9\tau}{2(4v-7\tau)} & \textit{if } 0 < \tau \leq \frac{4v}{13} \\ \frac{4v-5\tau}{4v+11\tau} & \textit{if } \frac{4v}{13} < \tau \leq \frac{4v}{5} \end{cases} \qquad (\textit{fixed prices}),$$

$$\overline{\delta}_T \begin{cases} = \frac{2(v-2\tau)}{4v+23\tau-13\sqrt{5}\tau} & \textit{if } 0 < \tau \leq \frac{v}{4} \\ \geq \frac{2(10\tau-v)(v-\tau)^2}{-2v^3+24v^2\tau+12v\tau^2+749\tau^3-351\sqrt{5}\tau^3} & \textit{if } \frac{v}{4} < \tau \leq \frac{4v}{5} \end{cases} \qquad (\textit{two-part tariff}).$$

The comparison of the critical discount factors reveals how the different pricing regimes affect the sustainability of (full) collusion. Figure 1 plots the critical discount factors for the case in which $v = 1$ against the degree of product differentiation. The two bottom lines refer to the cases of linear prices and fixed fees. The shaded area depicts the area where the critical discount factor in the case of two-part tariffs is located. The lower bound of this area is the bound derived in Lemma 3. This bound is sufficient to compare the three critical discount factors derived above. The upper bound $\overline{\delta}_N$ will be of interest later when we discuss fully nonlinear tariffs in Section 5.2.

Panel A: Competitive profits				
Type	Linear price	Fixed fee	Profit	Customer surplus
Linear prices	$p_L^* = \tau$	–	$\pi_L^* = \frac{\tau}{2}$	$CS_L^* = v - \tau$
Fixed fees	–	$f_F^* = \tau$	$\pi_F^* = \frac{\tau}{2}$	$CS_F^* = v - \frac{13\tau}{12}$
Two-part tariffs	$p_T^* = \frac{\tau(3\sqrt{5}-5)}{2}$	$f_T^* = \frac{\tau(7-3\sqrt{5})}{2}$	$\pi_T^* = \frac{\tau(13\sqrt{5}-27)}{4}$	$CS_T^* = v - \frac{\tau(23\sqrt{5}-45)}{6}$
Relevant comparisons for the understanding of Proposition 1: $\pi_T^* > \pi_L^* > \pi_F^*$				

Panel B: Collusive profits				
Type	Linear price	Fixed fee	Profit	Customer surplus
Linear prices	$p_L^c = v$	–	$\pi_L^c = \frac{v}{2}$	$CS_L^c = 0$
Fixed fees	–	$f_F^c = v - \frac{\tau}{4}$	$\pi_F^c = \frac{v}{2} - \frac{\tau}{8}$	$CS_F^c = \frac{\tau}{6}$
Two-part tariffs	$p_T^c = v$	$f_T^c = 0$	$\pi_T^c = \frac{v}{2}$	$CS_T^c = 0$
Relevant comparisons for the understanding of Proposition 1: $\pi_L^c = \pi_T^c > \pi_F^c$				

Panel C: Deviation profits			
Type	Linear price	Fixed fee	Profit
Linear prices	$p_L^d = \frac{2v-4\tau+A}{3}$	–	$\pi_L^d = \frac{(-2v+4\tau-A)(v^2-vA-4v\tau+2\tau A-20\tau^2)}{108\tau^2}$
Fixed fees	–	$f_F^d = v - \frac{5\tau}{4}$ if $0 < \tau \leq \frac{4v}{13}$ $f_F^d = \frac{v}{2} + \frac{3\tau}{8}$ otherwise	$\pi_F^d = v - \frac{5\tau}{4}$ if $0 < \tau \leq \frac{4v}{13}$ $\pi_F^d = \frac{(4v+3\tau)^2}{128\tau}$ otherwise
Two-part tariffs	$p_T^d = v - 2\tau$ if $0 < \tau \leq \frac{v}{4}$ p_T^d undefined otherwise	$f_T^d = \tau$ if $0 < \tau \leq \frac{v}{4}$ f_T^d undefined otherwise	$\pi_T^d = v - \tau$ if $0 < \tau \leq \frac{v}{4}$ $\pi_T^d \geq \frac{-v^3+12v^2\tau+6v\tau^2+10\tau^3}{54\tau^2}$ otherwise
Relevant comparison for the understanding of Proposition 1: $\pi_T^d > \pi_F^d$			

Table 1: Summary of results.

Figure 1 shows that collusion is most difficult to sustain under two-part tariffs and the comparison for linear prices and fixed fees is ambiguous. The following proposition states that this result is independent of the basic valuation v:

Proposition 1. *A comparison of the critical discount factors gives:*

1. *Collusion is less likely under two-part tariffs than under linear prices and fixed fees, that is,* $\overline{\delta}_T \geq \overline{\delta}_L$

and $\overline{\delta}_T \geq \overline{\delta}_F$.

2. *Collusion is less likely under fixed prices (linear prices) than under linear prices (fixed prices) for relatively low (high) degrees of product differentiation, that is,* $\tau^{(1)}$ *exists* ($\approx 0.5293500486v$)*, such that* $\overline{\delta}_L(\tau) < \overline{\delta}_F(\tau)$ *for* $\tau < \tau^{(1)}$ *and* $\overline{\delta}_L(\tau) > \overline{\delta}_F(\tau)$ *for* $\tau > \tau^{(1)}$.

To understand this result, we compare the different profits that determine the critical discount factors (see Table 1). Comparing competitive and deviation profits, we find that competition is less harsh and deviation is more profitable with two-part tariffs than with linear or fixed prices. This makes it harder to sustain collusion in the case of two-part tariffs. With linear prices, the collusive profits are identical to those with two-part tariffs, and, hence, the critical discount factor is lower. For the case of fixed fees, the collusive profits are lower, which means that there is an opposing effect that makes it more difficult to sustain collusion. However, as Proposition 1 states, this destabilizing effect is strictly dominated by the aforementioned facilitating effects.

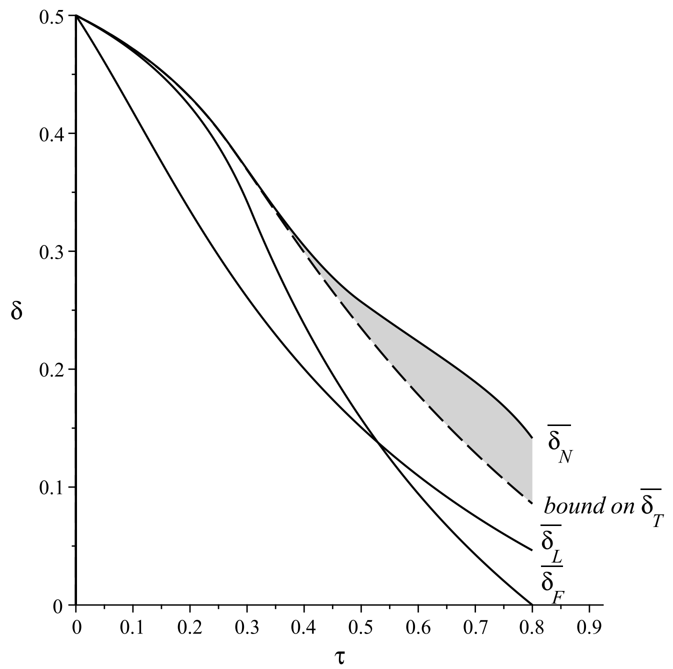

Figure 1: Comparison of the critical discount factors in the three scenarios (for $v = 1$ and $0 < \tau \leq 4/5$).

Comparing fixed to linear prices, we point out that both pricing schemes yield the same competitive profits. However, linear prices have two opposing effects with regard to the likelihood of collusion: On the one hand, collusive profits are higher, which makes collusion more attractive for firms. On the other hand, higher collusive prices make deviation easier, which results in higher deviation profits. For moderately differentiated products (that is, $\tau < \tau^{(1)}$), deviation proves attractive in the case of fixed fees, which means that the first effect dominates, and collusion is easier to sustain with linear prices. The reason is that with fixed fees, customers do not mix and, hence, a deviating firm has to compensate the customers for their transport costs. This compensation is relatively cheap when the degree of product differentiation is low. When product differentiation increases, compensating the customers becomes more costly and the second effect becomes more important (that is, for $\tau > \tau^{(1)}$).

5. Robustness checks

In this section, we will investigate to what extent our results rely on the assumptions imposed in the two previous sections. In doing so, we will first focus on the structure of firms' pricing schedules and extend our model to capture more flexible pricing structures (so-called "fully nonlinear tariffs"). Second, we will relax the assumption that firms collude only on profit-maximizing prices. Before we discuss these two aspects in more detail, however, a more general remark with regard to the comparison of the likelihood of collusion under linear prices and two-part tariffs seems to be in order.

5.1. An initial remark on likelihood of collusion under linear prices and two-part tariffs

In our baseline analysis, we assume that firms use grim-trigger strategies, that is, a firm will set the competitive price in all future periods once it observes that its competitor deviates from a collusive agreement. Although grim-trigger strategies are themselves an assumption, their application may shed additional light on the comparison of the stability of collusive agreements under linear prices and two-part tariffs.

As shown in the previous section, the linear price component is an important tool for colluding firms to extract customers' surplus. Its importance is so large that colluding firms use only linear prices and dismiss the possibility to set a strictly positive fixed fee even if they may use two-part tariffs in general. It is easy to think of many modifications to our model that leave this result intact. For instance, if customers are not uniformly distributed, but are distributed according to any other distribution, the decision to focus only on linear prices is still optimal in the sense that it maximizes (joint) collusive profits. The reason is that by setting equal linear prices, firms allow each customer—regardless of the location—to obtain the optimal combination, which in turn maximizes welfare. At the same time, firms are able to extract the entire utility from each customer, so that they obtain the highest possible profit.

This has in turn implications for the optimal prices of a deviating firm. The deviation profits must be (weakly) larger if a firm has access to an additional pricing instrument. If the additional instrument is not useful, the firm will simply not use it and set it equal to zero. Therefore, deviation profits must be (weakly) larger with two-part tariffs than with linear prices whenever it is optimal for colluding firms to focus only on linear prices.

We believe that the above argumentation that collusive profits are the same under both pricing regimes and deviation profits are (weakly) larger under two-part tariffs applies to various modifications. With grim-trigger strategies, the critical discount factor depends on three different types of profits. The above results for the collusive and deviation profits point to lower stability of collusive agreements with two-part tariffs than with linear prices. The only opposing force that could lead to a lower likelihood of collusion under linear prices could stem from the competitive profits. However, the intuition behind how firms use the fixed fee component in the case of two-part tariffs suggests that competitive profits are unlikely to be higher under linear prices than under two-part tariffs. The reason is that by setting a moderate fixed fee, firms can exploit loyal customers. Customers located in the middle of the interval can save on their transport costs by buying from both firms, and as long as the magnitude of the fixed fee is moderate (relative to the transport costs), these customers will prefer to buy from both firms. The customers a firm loses are those who are located close to the competitor and would buy only small shares under linear prices anyway. Therefore, the fixed fee component appears to be a beneficial additional tool that is likely to increase rather than decrease competitive profits.

Following this line of reasoning, we conjecture that our result that collusion is more stable under linear prices than under two-part tariffs is robust to many modifications (for instance, to different distributional assumptions).

5.2. Fully nonlinear pricing schemes

We now turn our focus to the structure of tariffs used in the baseline analysis. So far, we have focused on three pricing schemes that are common in the real world and often used in economic analyses. However, this limitation may raise the concern of whether our results extend to other pricing schemes. In particular, it is of interest how larger flexibility in the pricing structure might affect our results. To address this issue, we introduce fully nonlinear tariffs along the lines of Hoernig and Valletti (2011).

A fully nonlinear tariff is a function $T : [0, 1] \rightarrow \mathbb{R}$ that assigns a price $p(q)$ to each possible quantity $q \in [0, 1]$. This tariff structure incorporates all other possible pricing schemes. For example, a linear price p is equal to $T(q) = pq$, a fixed fee f to $T(q) = f$ and a two-part tariff (p, f) to $T(q) = pq + f$.

Hoernig and Valletti (2011) base their analysis on two sets of assumptions that we adopt as well. First, they require the tariffs to be differentiable on $(0, 1)$, but do not impose any assumption on the endpoints of the interval. Second, they allow each customer to buy more than one unit in total and restrict the information set of the retailers. More precisely, the authors assume that each firm can only observe the quantity that a customer buys from itself, but not the quantity bought from its competitor or the total quantity purchased. Thus, in combination with the assumption on customer behavior, it cannot expect that a customer who buys a share of λ from itself will automatically buy $1 - \lambda$ from its competitor. Although each customer ends up buying only one unit in total, customers may buy more than one unit if it is cheaper (and throw away the additional quantity purchased). Therefore, this assumption rules out various forms of contractual relationship, including exclusive dealing, quantity forcing, and market-share contracts.

Based on these assumptions, Hoernig and Valletti (2011) analyze competitive outcomes and prove the existence of a unique Nash equilibrium:

Recap 2. *If firms compete in fully nonlinear tariffs, they set*

$$T_N^*(q) = \tau \cdot q + \frac{\tau \cdot q \cdot (1-q)}{3}$$

and earn profits of $\pi_N^* = 14\tau/27$.

With this result in hand, we only need to calculate the collusive and deviation profits to derive the critical discount factor. Based on our previous analysis, it is straightforward to derive the profit-maximizing collusive prices. As argued before, a linear collusive price of v leads to the largest possible profit because it maximizes welfare and allows firms to extract all rents, so that customer surplus is zero. Linear prices can be expressed as fully nonlinear tariffs, which leads to the following result:

Corollary 1. *Profit-maximizing collusive tariffs are given by* $T(q) = vq$*, and firms earn profits of* $\pi_N^c = v/2$.

This leaves us with the question of the optimal deviation strategy. The following lemma specifies the profit of a deviating firm and the resulting critical discount factor:

Lemma 4. *With fully nonlinear tariffs, the optimal deviation profits are given by:*

$$\pi_N^d = \begin{cases} v - \tau & \text{if } \tau \leq \frac{v}{4} \\ \frac{16\tau^3+12\tau v^2-v^3}{48\tau^2} & \text{if } \frac{v}{4} < \tau < \frac{(3-\sqrt{6})v}{2} \\ \frac{99\sqrt{6}v^3-243v^3+8\tau^3+6v^2\tau}{24\tau^2} & \text{if } \tau \geq \frac{(3-\sqrt{6})v}{2}. \end{cases}$$

Let $C := -360\left(\frac{4\tau}{5}+v\right)\sqrt{2}\,v\sqrt{\tau^2 v\,(2\tau+v)} + 513\tau v^3 + 864\tau^2 v^2$. *The critical discount factor is*

$$\bar{\delta}_N = \begin{cases} \frac{54\tau-27v}{82\tau-54v} & \text{if } \tau \leq \frac{v}{4} \\ \frac{144\tau^3+108\tau v^2-216\tau^2 v-9v^3}{108\tau v^2-80\tau^3-9v^3} & \text{if } \frac{v}{4} < \tau < \frac{v}{2} \\ \frac{C+216\tau^3 v}{C+224\tau^4} & \text{if } \tau \geq \frac{v}{2}. \end{cases}$$

As in the case of two-part tariffs, three cases characterize the optimal deviation strategy. If product differentiation is sufficiently low, a deviating firm prefers to monopolize the market because it is rather cheap to compensate customers located close to its competitor for their transport costs. With an increasing degree of product differentiation, this compensation becomes too costly. Therefore, an intermediate level of product differentiation enables customers close to the competitor to buy from both firms, thereby reducing their transport costs. At a high level of product differentiation, the deviating firm completely abandons these customers.

In Figure 1, the line for the critical discount factor in the case of fully nonlinear tariffs reveals that the critical discount factor is largest among all four pricing schemes. The following proposition states that this finding is independent of the parameterization used for the figure:

Proposition 2. *The critical discount factor is largest in the case of fully nonlinear tariffs, that is,* $\overline{\delta}_N > \overline{\delta}_t$ *with* $t \in \{L, F, T\}$.

Our previous analysis showed that collusion is least likely under two-part tariffs compared to linear prices or fixed fees. To understand Proposition 2, it is therefore sufficient to compare fully nonlinear tariffs to two-part tariffs. Our analysis reveals that under both pricing regimes, firms use the same collusive prices and obtain the same profits. A deviating firm can therefore only benefit from the more flexible pricing structure in the case of fully nonlinear tariffs. To see this, note that this pricing structure also incorporates two-part tariffs. Thus, a deviating firm could simply use the optimal two-part tariff if this was the best strategy overall. Our analysis indicates that this is indeed true for low degrees of product differentiation because it monopolizes the entire market under both pricing regimes and the costs of compensating the customer located at $x = 1$ are independent of the pricing schemes analyzed. However, once the deviating firm does not want to monopolize the entire market, the nonlinear tariffs make it easier to compensate customers located far away. The reason is that some customers combine products of both firms and, depending on their location, can reduce their transport costs to different extents. The fully nonlinear tariff allows the deviating firm to better adjust to these differences in utility levels and to better target individual customers (and, thus, better extract each customer's surplus). In summary, deviating becomes more profitable, which in turn destabilizes collusion. Moreover, we know that the competitive profits are larger under fully nonlinear tariffs than under two-part tariffs, which also leads to lower stability of collusion. We can thus conclude that the insights from the baseline set-up hold under fully nonlinear tariffs.

5.3. Partial collusion

In our baseline analysis, we considered firms' ability to collude on profit-maximizing prices. In the context of the Hotelling (1929) framework, Chang (1991) shows that if collusion on these prices is not sustainable, firms can still collude by setting prices below the profit-maximizing collusive but above the competitive prices.[20] We refer to this behavior as partial collusion. In this section, we adopt idea of Chang (1991) and

[20] In Chang (1991), firms have only access to one price instrument (a per-unit price). With two-part tariffs, the framing that firms collude on "prices below the profit-maximizing prices" might be wrong. For instance, a firm might find it beneficial to decrease the linear price and to increase the fixed fee to stabilize a collusive agreement. Therefore, when we refer to two-part tariffs, we will talk about prices that are *different* from the profit-maximizing prices.

discuss whether and how our results change when firms collude on prices different from the profit-maximizing prices.

We start with the case of fixed fees. We already know from the previous analysis that firms do not have an incentive to set prices so low that (some) customers buy from both firms. So far, this was true for all cases (competition, collusion, and deviation). Given this result, it is not surprising that the same is true for partial collusion. If all customers decide to buy exclusively from one firm, we face the same game as in Chang (1991) and get the same result:

Corollary 2. *Assume that firms' pricing is restricted to fixed fees only and that collusion at profit-maximizing prices is not sustainable (that is, $\delta < \overline{\delta}_F$). Optimal collusive prices are given by*

$$f_F^c(\delta) = \begin{cases} \frac{\tau(2-3\delta)}{1-2\delta} & \text{if } \delta > \frac{1}{3} \\ \frac{\tau(1+3\delta)}{1-\delta} & \text{if } \delta \leq \frac{1}{3}, \end{cases}$$

and the profit is $\pi_F^c(\delta) = f_F^c(\delta)/2$.

Turning to linear prices, our previous analysis has shown that it is never profitable for the deviating firm to monopolize the market. This also remains true for the case of partial collusion because the deviating firm now faces an even lower collusive price set by the rival. Thus, it always leaves its rival with a strictly positive market share. We obtain the following characterization of the optimal linear prices:

Lemma 5. *Assume that firms' pricing is restricted to linear prices only and that collusion at profit-maximizing prices is not sustainable (that is, $\delta < \overline{\delta}_L$). The optimal collusive price $p_L^c(\delta)$ is implicitly defined by*

$$\delta = \frac{\left(-28\tau^2 + 4\tau p_L^c - {p_L^c}^2\right)\sqrt{28\tau^2 - 4\tau p_L^c + {p_L^c}^2} + 136\tau^3 - 6p_L^c\tau^2 - 6\tau\, {p_L^c}^2 + {p_L^c}^3}{\left(-28\tau^2 + 4\tau p_L^c - {p_L^c}^2\right)\sqrt{28\tau^2 - 4\tau p_L^c + {p_L^c}^2} + 190\tau^3 - 60p_L^c\tau^2 - 6\tau\, {p_L^c}^2 + {p_L^c}^3},$$

and the profit is $\pi_L^c(\delta) = p_L^c(\delta)/2$. The optimal collusive price $p_L^c(\delta)$ is strictly decreasing in the discount factor $\delta > 0$.

With the results for linear and fixed prices in hand, we can compare the profits from both pricing schemes for any given discount factor:

Proposition 3. *If firms collude, firms always gain larger profits with linear prices than with fixed fees, that is, $\pi_L^c(\delta) > \pi_F^c(\delta)$ for all $\delta > 0$.*

To understand this result, it is useful to focus on the impact of product differentiation on colluding firms' price setting. We start with a relatively small degree of product differentiation. We know from the previous analysis that in this case, collusion on profit-maximizing prices is sustainable for a larger range of discount factors with linear prices than with fixed prices. This is because with fixed prices, it is cheap for the deviating firm to monopolize the entire market, which results in relatively large deviation profits. When we extend this idea to the current set-up with partial collusion, this likely means that with fixed fees, firms must lower the collusive fee substantially in order to render deviation unprofitable. Combined with the fact that even if firms are able to collude on profit-maximizing prices, firms obtain larger collusive profits with linear prices than with fixed fees, it is not surprising that firms gain larger (partially) collusive profits with linear prices.

Turning the focus to relatively large degrees of product differentiation, deviation becomes less attractive in the case of fixed fees. The reason is that the deviating firm has to compensate the indifferent customer for the transport costs to induce this customer to switch, and this compensation comes increasingly costly as the degree of product differentiation increases. This also leads to the result that collusion on profit-maximizing prices is sustainable for a larger range of the discount factor with fixed fees than with linear prices. If deviation becomes less attractive with fixed prices, the extent to which (partially) colluding firms have to lower the collusive price to render deviation unprofitable decreases as well. This positively affects collusive profits with fixed prices. There is, however, an opposing channel that is caused by a large degree of product differentiation. As noted above, even if firms are able to collude on profit-maximizing prices, firms obtain larger collusive profits with linear prices than with fixed fees. This difference in profits increases in the degree of product differentiation. The reason is that with symmetric linear prices, customers always buy their optimal mix, so that transport costs are zero. Thus, the surplus that colluding firms can extract is independent of the degree of product differentiation. By contrast, with fixed fees, customers do not mix and, hence, the total surplus that firms can extract decreases in the degree of product differentiation. Proposition 3 shows that the first positive effect of higher transport costs does not outweigh this second effect.

In Section 4, where we consider collusion on profit-maximizing prices, we use a bound on the critical discount factor for two-part tariffs because it is difficult to derive a closed-form solution. Not surprisingly, we run into similar problems with the analysis of partial collusion. Therefore, we base the remaining investigation of partial collusion on numerical simulations.[21] In each simulation, we fix the set of exogenous parameters (that is, the basic valuation and the transport cost parameter) and identify the firms' optimal collusive behavior for discount factors between 0.01 and 0.5. For discount factors larger than 0.5, firms can collude on profit-maximizing prices in any of the three pricing scenarios (see also Figure 1).

For linear prices and fixed fees, we can calculate the outcomes using the results above, so our simulation concerns only the case of two-part tariffs. In the first step of our simulation, we run a "brute force" procedure and go through all possible collusive (linear and fixed) prices with precision 0.01. For a given collusive two-part tariff, we search for the optimal deviation response of the competitor.

[21]Note that we do not claim that a solution does not exist, but such a solution is simply too difficult to derive because of the highly nonlinear relationships between prices and the discount factor.

This approach gives us information about the optimal deviation strategies for a given collusive two-part tariff. In the next step, we evaluate which collusive tariff is optimal for a given critical discount factor. The discount factors are arranged on a grid with precision 0.01. For each discount factor, we loop over all possible collusive tariffs. For each candidate tariff, we can calculate the collusive profit and extract the deviation profit from the "brute force" procedure. The competitive behavior is not affected by the candidate tariff, and the competitive profit results from the static Nash equilibrium. First, we check whether the critical discount factor that we can calculate based on Expression (2) is below the currently considered discount factor. Among the tariffs that satisfy this necessary condition, we then pick the tariff that yields the highest collusive profit.

We run our simulation for three different parameter configurations that refer to different degrees of product differentiation. Note that both the basic valuation v and the transport cost parameter τ can be used to model product differentiation. For example, to investigate an increase in product differentiation, we can either decrease the basic valuation or increase the transport cost parameter (ceteris paribus). We therefore fix the basic valuation at $v = 1$ and only vary the transport costs. More specifically, Assumption 1 requires that the highest value for the transport cost parameter is given by 0.8 and we use 10%, 50%, and 90% of this upper bound to discuss the cases of relatively small, intermediate, and large product differentiation. The figures presented in the text refer to the intermediate set-up with $\tau = 0.4$. The other figures can be found in Appendix B.

The main objective of the simulation is to understand how the outcomes with two-part tariffs compare to the outcomes with linear and fixed prices. For the comparison of linear prices and two-part tariffs, we can make an initial hypothesis. We know from Section 4 that full collusion (that is, on profit-maximizing prices) is feasible for a larger range of discount factors with linear prices. Because the collusive profits with profit-maximizing prices are the same for both pricing regimes, this means that there is a range of discount factors for which firms using two-part tariffs must deviate from the profit-maximizing two-part tariffs, whereas firms using linear prices can sustain the profit-maximizing prices. In this range, profits are larger with linear prices than with two-part tariffs. On the other hand, we know from other research (for example, Chang, 1991) that collusive profits approach competitive profits if the discount factor tends to zero. Because competitive profits are larger with two-part tariffs than with linear prices, we can expect the profit curves for linear prices and two-part tariffs to intersect at least once. Profits should be larger with two-part tariffs for very small discount factors and larger with linear prices for discount factors sufficiently close to the critical discount factors from Section 4.

Our simulation lends support to this hypothesis. Figure 2 plots profits and customer surpluses against discount factors for the different pricing schemes. It illustrates that collusive profits are largest and customer surplus is lowest in the case of linear prices if the discount factor is not extremely small. If the discount factor tends to zero, collusive profits approach competitive profits and, hence, collusive profits are largest with two-part tariffs (i.e., they approach $\pi_L^* = \pi_F^* = 0.2$ and $\pi_T^* \approx 0.207$).

The figure further reveals that collusive profits are always lowest with fixed fees. Customer surplus is largest when collusion on profit-maximizing fixed fees is sustainable. When firms have to collude partially on fixed fees, customer surplus is similar to that in the case of two-part tariffs. Consequently, customer surplus is also smaller with fixed fees than with linear prices if discount factors are sufficiently small. The reason is that collusive profits tend to competitive profits that are equal under both pricing schemes. In other words, firms extract roughly the same part of customers' utility if discount factors tend to zero. At the same time, customers do not mix and, hence, incur transport costs when firms set fixed fees instead of linear prices.

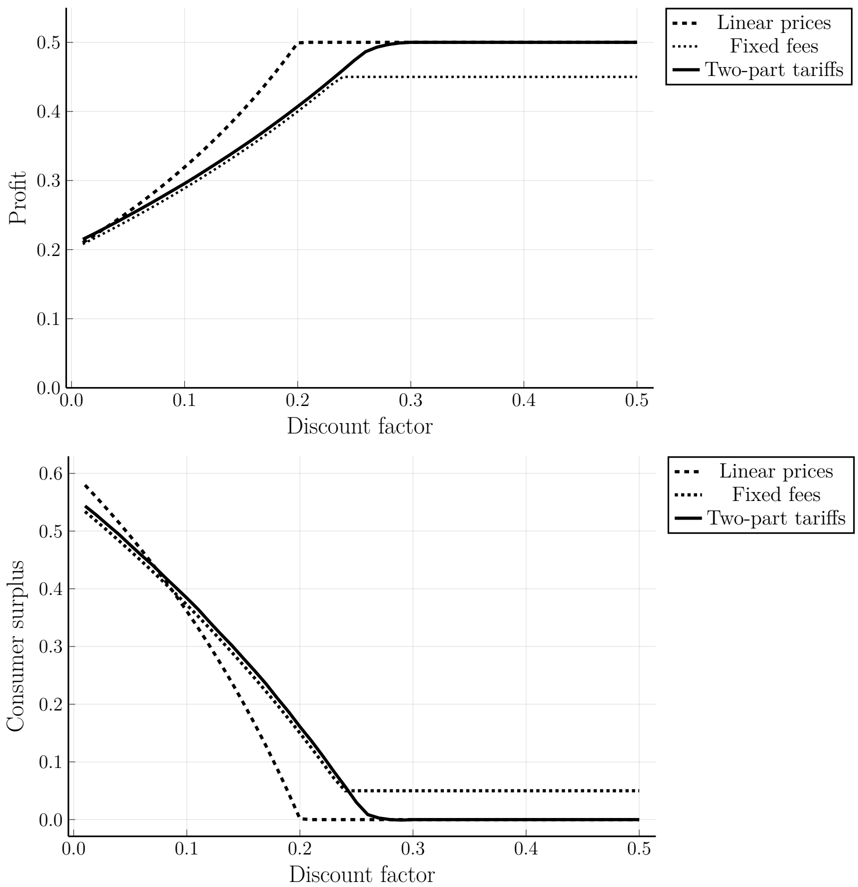

Figure 2: Profits and customer surpluses under partial collusion ($v = 1$, $\tau = 0.4$).

The corresponding figures for the cases of low and high product differentiation are Figures B.3 and B.4. Both figures qualitatively support our findings, although the quantitative measures, such as thresholds and distances, vary.

To put these results in perspective, remember that Proposition 1 shows that simple (that is, single-part) pricing schedules make it easier for firms to coordinate on profit-maximizing prices. The main insight of this subsection is that firms are likely to benefit from a ban of the fixed price component of a two-part tariff – even if they can only sustain collusion below the critical discount factor. At the same time, the ban is likely to harm customers most. Again, this finding is especially important because the ban is most beneficial for customers in absence of collusion.[22] We also obtain new insights with regard to the ban of the linear price component. When we compare fixed prices to two-part tariffs, we find that the ban could facilitate collusion on the one hand, but on the other hand, it can be beneficial for customers if firms partially collude anyway.

6. Summary

This paper investigates firms' incentives to collude in a framework in which customers have the ability to combine products from different firms to achieve a better fit of their preferences. Motivated by various examples from the banking and insurance industry, we consider two policy interventions (banning linear or fixed prices) and investigate the case of partial collusion in Section 5.3. Table 2 summarizes our findings. First, firms can be restricted to use linear prices only, which leads to an increase in customer surplus in a static environment. However, it is shown that such a restriction makes it easier for firms to collude and harms customers. Additionally, our investigation of partial collusion shows that in the presence of collusion, linear prices are again most likely to lead to the highest profits and the lowest customer surplus among all pricing schemes. In summary, we conclude that the possibility to have higher customer surplus in absence of collusion comes along with an increasing scope for collusion and lower customer surplus in the presence of collusion.

Second, we consider a ban of linear prices, so that firms must compete (or collude) with fixed prices. Although collusion on profit-maximizing prices is easier with fixed prices than with two-part tariffs, we find that firms prefer to partially collude with two-part tariffs and fixed fees can harm customers the least among the three collusive pricing regimes. In summary, fixed fees can be less harmful to customers in presence of collusion, whereas they are most harmful to customers in absence of collusion (Hoernig and Valletti, 2007).

In summary, the present analysis has important implications for competition and consumer protection policy. The previous literature has shown that customers can benefit from policy interventions. Our paper highlights that such interventions can have undesired consequences and the implications are thus ambiguous: The possibility to achieve a higher customer surplus in absence of collusion may come at the expense of an increasing scope for collusion. Even in the worst case in which firms always collude, less price instruments can result in lower customer surplus and, hence, harm customers.

[22] Note that due to the robustness of the result, it does not matter for the policy maker whether the actual industry discount factor is known. This proves to be quite convenient because generally speaking, little is known about real-life discount factors. A notable exception is Igami and Sugaya (2021) who analyze the vitamin cartels.

	Competition	Collusion
Critical discount factor	–	$\overline{\delta}_L < \overline{\delta}_F < \overline{\delta}_T$ if $\tau < \tau^{(1)}$ $\overline{\delta}_F < \overline{\delta}_L < \overline{\delta}_T$ if $\tau > \tau^{(1)}$
Producer surplus	$\pi_F^* = \pi_L^* < \pi_T^*$	$\pi_F^c < \pi_T^c = \pi_L^c$
Customer surplus	$CS_F^* < CS_T^* < CS_L^*$	$CS_T^c = CS_L^c < CS_F^c$
Social welfare	$W_F^* < W_T^* < W_L^*$	$W_F^c < W_T^c = W_L^c$

Table 2: Comparison of critical discount factors, profits, customer surpluses, and welfare.

Finally, as with every (theoretical) analysis, our analysis is limited by the assumptions of our model and the usual caveats apply. The assumptions of unit demand and the uniform distribution of consumers along the linear city are just two examples. However, in light of our general argument on the comparison of the price-setting incentives under linear prices and two-part tariffs (Section 5.1), we are confident that our main result remains intact even if one relaxes these assumptions.

References

Abreu, D., 1986. Extremal equilibria of oligopolistic supergames. Journal of Economic Theory 39, 191–225.

Abreu, D., 1988. On the theory of infinitely repeated games with discounting. Econometrica 56, 383–396.

Abreu, D., Pearce, D., Stacchetti, E., 1986. Optimal cartel equilibria with imperfect monitoring. Journal of Economic Theory 39, 251–269.

Anderson, S.P., Neven, D.J., 1989. Market efficiency with combinable products. European Economic Review 33, 707–719.

Bain & Company, 2012. Was Versicherungskunden wirklich wollen.

Bos, I., Marini, M.A., 2019. Cartel stability under quality differentiation. Economics Letters 174, 70–73.

Bos, I., Marini, M.A., Saulle, R.D., 2020. Cartel formation with quality differentiation. Mathematical Social Sciences 106, 36–50.

Chang, M.H., 1991. The effects of product differentiation on collusive pricing. International Journal of Industrial Organization 9, 453–469.

Chang, M.H., 1992. Intertemporal product choice and its effects on collusive firm behavior. International Economic Review 33, 773–793.

Chen, H.C., Ritter, J.R., 2000. The seven percent solution. Journal of Finance 55, 1105–1131.

Colombo, S., 2010. Product differentiation, price discrimination and collusion. Research in Economics 64, 18–27.

Colombo, S., Pignataro, A., 2022. Information accuracy and collusion. Journal of Economics & Management Strategy 31, 638–656.

d'Aspremont, C., Gabszewicz, J.J., Thisse, J.F., 1979. On Hotelling's "Stability in competition". Econometrica 47, 1145–1150.

Friedman, J.W., 1971. A non-cooperative equilibrium for supergames. Review of Economic Studies 38, 1–12.

Gabszewicz, J.J., Laussel, D., Sonnac, N., 2004. Programming and advertising competition in the broadcasting industry. Journal of Economics & Management Strategy 13, 657–669.

Gal-Or, E., Dukes, A., 2003. Minimum differentiation in commercial media markets. Journal of Economics & Management Strategy 12, 291–325.

Gössl, F., Rasch, A., 2020. Collusion under different pricing schemes. Journal of Economics & Management Strategy 29, 910–931.

Häckner, J., 1994. Collusive pricing in markets for vertically differentiated products. International Journal of Industrial Organization 12, 155–177.

Häckner, J., 1995. Endogenous product design in an infinitely repeated game. International Journal of Industrial Organization 13, 277–299.

Häckner, J., 1996. Optimal symmetric punishments in a bertrand differentiated products duopoly. International Journal of Industrial Organization 14, 611–630.

Hansen, R.S., 2001. Do investment banks compete in ipos?: the advent of the "7% plus contract". Journal of Financial Economics 59, 313–346.

Helfrich, M., Herweg, F., 2016. Fighting collusion by permitting price discrimination. Economics Letters 145, 148–151.

Hoernig, S.H., Valletti, T.M., 2007. Mixing goods with two-part tariffs. European Economic Review 51, 1733–1750.

Hoernig, S.H., Valletti, T.M., 2011. When two-part tariffs are not enough: Mixing with nonlinear pricing. B.E. Journal of Theoretical Economics 11.

Horstmann, N., Krämer, J., 2013. Price discrimination or uniform pricing: Which colludes more? Economics Letters 120, 379–383.

Hotelling, H., 1929. Stability in competition. Economic Journal 39, 41–57.

Igami, M., Sugaya, T., 2021. Measuring the uncentive to collude: The vitamin cartels, 1990–99. Review of Economic Studies 89, 1460–1494.

Jorge, S.F., Pires, C.P., 2008. Delivered versus mill nonlinear pricing with endogenous market structure. International Journal of Industrial Organization 26, 829–845.

Liu, Q., Serfes, K., 2007. Market segmentation and collusive behavior. International Journal of Industrial Organization 25, 355–378.

Matutes, C., Regibeau, P., 1992. Compatibility and bundling of complementary goods in a duopoly. Journal of Industrial Economics 40, 37–54.

Miklós-Thal, J., 2008. Delivered pricing and the impact of spatial differentiation on cartel stability. International Journal of Industrial Organization 26, 1365–1380.

OECD, 1998. Competition and Related Regulation Issues in the Insurance Industry.

Peiseler, F., Rasch, A., Shekhar, S., 2022. Imperfect information, algorithmic price discrimination, and collusion. Scandinavian Journal of Economics 124, 516–549.

Ross, T.W., 1992. Cartel stability and product differentiation. International Journal of Industrial Organization 10, 1–13.

Symeonidis, G., 1999. Cartel stability in advertising-intensive and R&D-intensive industries. Economics Letters 62, 121–129.

Appendix A. Proofs

Proof (Lemma 1).

Linear prices:

The case of collusion with linear prices corresponds to the monopoly case in Hoernig and Valletti (2011), which they use as a benchmark for their analysis. The authors assume that a monopolist offers both products that are located at the extreme points of the line. This is the same optimization problem as in the case of two colluding firms which are located at the same extreme points. The firms – just like the monopolist – have an incentive to maximize welfare (that is, induce the efficient allocation where $\underline{x} = 0$ and $\overline{x} = 1$) which is then fully extracted by setting an optimal linear price of v.

Fixed fees:

First, we assume that firms set prices in such a way that at least some customers buy from both firms. We thus have:

$$\underline{x} \leq \overline{x} \quad \Leftrightarrow \quad \sqrt{\frac{f_2}{\tau}} \leq 1 - \sqrt{\frac{f_1}{\tau}}$$

Note that if the constraint binds, the indifferent customer who is located at $x = \underline{x} = \overline{x}$ is indifferent between buying from both firms or buying from one firm exclusively.

Both firms maximize their joint profit that is given by

$$\Pi = f_1 \cdot \overline{x} + f_2 \cdot (1 - \underline{x}) \, .$$

We calculate the derivative of the profit function with respect to both fixed prices:

$$\frac{\partial \Pi}{\partial f_1} = 1 - \frac{3}{2}\sqrt{\frac{f_1}{\tau}}$$
$$\frac{\partial \Pi}{\partial f_2} = 1 - \frac{3}{2}\sqrt{\frac{f_2}{\tau}}$$

When we set the derivatives to zero, we find the candidate solution $f_1 = f_2 = 4\tau/9$. Evaluating the aforementioned constraint at these values yields $2/3 \leq 1/3$, which is obviously not true. Because the first derivatives are always larger than zero for $0 \leq f_1, f_2 \leq 4\tau/9$, we conclude that, given that the aforementioned assumption has to be satisfied, firms prefer to set their prices in a way such that the constraint binds and the indifferent customer is indifferent between combining both products or buying from one firm exclusively. Because the decision of the indifferent customer has no influence on the joint profit, it is sufficient to analyze the case in which all customers buy exclusively from one firm.

Now we turn to the case in which customers do not mix. Given Assumption 1, firms set prices, such that all customers have non-negative utilities and all customers buy. Thus, total demand is not affected by the price level and firms set prices, such that the customer who is indifferent between buying from firm 1 and buying from firm 2 is also indifferent between buying and not buying. Let x be the position of this customer. Then, firm 1 sets $v - \tau \cdot x^2$ and realizes a market share of x. Analogously, firm 2 sets $v - \tau \cdot (1-x)^2$ and realizes a market share of $1 - x$. The joint profit is given by Π:

$$\Pi = x \cdot \left[v - \tau \cdot x^2\right] + (1-x) \cdot \left[v - \tau \cdot (1-x)^2\right]$$
$$\frac{\partial \Pi}{\partial x} = -3 \cdot \tau \cdot x^2 + 3 \cdot \tau \cdot (1-x)^2$$
$$\frac{\partial^2 \Pi}{\partial x^2} = -6 \cdot \tau \left(x + (1-x)\right) < 0$$

The first derivative is zero if $x = 1/2$. Therefore, the optimal collusive price equals $v - t/4$.

Two-part tariffs:

Customers do not buy if they have to pay more than the reservation price v. Therefore, the highest possible profit is v. Note that if both firms charge no fixed fees (that is, $f_i = 0$) and optimal linear prices (that is, $p_i = p_L^c = v$), the total profit is equal to v, that is, the profit is maximized.

We show that no other combination of fixed fees and linear prices is a collusive equilibrium. First note that if the fixed fee is zero, symmetric linear prices lower than v lead to a total profit lower than v and, hence, will not be chosen in equilibrium.

Note further that firms set symmetric linear prices. In the case of asymmetric linear prices, customers suffer from strictly positive transport costs and, hence, customers' total utility that firms can extract is lower than v.

Finally, we show that firms have no incentive to set strictly positive fixed fees. If at least one fixed fee was larger than zero, at least one indifferent customer ($\underline{x}$ and/or $\overline{x}$) is not located at the extreme, that is, at least some customers do not mix and suffer from strictly positive transport costs. As a result, customers' total utility that firms can extract is lower than v, and, hence, the prices will not be chosen in equilibrium.

Proof (Lemma 2).

We assume without loss of generality that firm 1 deviates from the collusive agreement.

Linear prices:

Firm 1 can set its price, such that it either monopolizes the market or firm 2 realizes a strictly positive market share. Note that customers do not mix in the first case (that is, $\underline{x} = \overline{x} = 1$), while there is a share of customers who mix in the latter case (that is, $\underline{x} < \overline{x} = 1$). We derive prices and payoffs for both cases. When firm 1 monopolizes the market, the highest possible price satisfies the constraint $\underline{x} = 1$. Thus, we find $p_L^{d_1} = \pi_L^{d_1} = v - 2\tau$. Otherwise, in the second case, we plug the collusive price, p_L^c, as the price of firm 2 into firm 1's profit function and maximize the profit with respect to firm 1's own price. This leads to

$$p_L^{d_2} = \frac{2v - 4\tau + A}{3}, \; \pi_L^{d_2} = \frac{(-2v + 4\tau - A)(v^2 - vA - 4v\tau + 2\tau A - 20\tau^2)}{108\tau^2}.$$

Note that $\underline{x}_L^{d_2} := \underline{x}_L\left(p_1 = p_L^{d_2}, p_2 = p_L^c\right) < 1$ holds for all $0 < \tau \le 4v/5$. Comparing profits $\pi_L^{d_1}$ and $\pi_L^{d_2}$, we find that sharing the market is always more profitable than monopolization. As a result, firm 1 sets $p_L^{d_2}$ and earns $\pi_L^{d_2}$.

Fixed fees:

Firm 1 can set its fixed price, such that it either monopolizes the market or firm 2 realizes a strictly positive market share. In the first case, firm 1 has to compensate the loyal customers of firm 2 for suffering from higher transport costs. The customer at location $x = 1$ suffers from the highest transport costs of τ. As a result, prices and profits are given by

$$f_F^{d_1} = \pi_F^{d_1} = f_F^c - \tau = v - \frac{5\tau}{4}.$$

Next, we analyze the case in which firm 1 does not monopolize the market. First assume that it sets its price f, such that at least some customers mix and, hence, buy from both firms. In this case, $\underline{x} < \overline{x}$ holds. Firm 2 sticks to the collusive price schedule and sets $f_F^c = v - \tau/4$ (see Lemma 1). We can plug the prices into the aforementioned inequality and obtain

$$\sqrt{\frac{v}{\tau} - \frac{1}{4}} < 1 - \sqrt{\frac{f}{\tau}}.$$

The right-hand side of the inequality is always smaller than or equal to one. At the same time, we can rearrange the left-hand side and obtain

$$\sqrt{\frac{v}{\tau} - \frac{1}{4}} \ge 1 \quad \Leftrightarrow \quad \frac{v}{\tau} - \frac{1}{4} \ge 1 \quad \Leftrightarrow \quad \frac{v}{\tau} \ge \frac{5}{4} \quad \Leftrightarrow \quad \frac{4}{5}v \ge \tau.$$

Assumption 1 ensures that the first equivalence sign is correct and the final inequality holds. Because the left-hand side is always larger than or equal to one, but the right-hand side is less than or equal to one, the initial inequality $\underline{x} < \overline{x}$ does not hold. In other words, it is not possible that the deviating firms sets a fixed fee, such that at least some customers buy from both firms.

Turning to the case in which customers do not mix, we plug the collusive price, f_F^c, into firm 1's profit function and maximize the profit with respect to firm 1's own price. The resulting price and profit are

$$f_F^{d_2} = \frac{v}{2} + \frac{3\tau}{8}, \quad \pi_F^{d_2} = \frac{(4v + 3\tau)^2}{128\tau}.$$

The second case is relevant if and only if the profit is larger than the profit in the first case and the customer who is indifferent between buying from firm 1 or firm 2 is located in the interval. Both conditions lead to $\tau > \frac{4v}{13}$.

Two-part tariffs:

We have to distinguish between three cases to calculate the deviation profits: (i) Firm 1 monopolizes the market, that is, $\underline{x} = \overline{x} = 1$, (ii) (at least some) customers buy from both firms and the entire market is served, and (iii) there are some customers who use the outside option and do not buy from any firm. Note that customers will never buy from firm 2 exclusively. This is because they will pay a price of v, which equals their valuation, and, in addition, suffer from the transport costs. This implies $\underline{x} < \overline{x} = 1$ for case (ii).

It is difficult to derive a (closed-form) solution for the third case. Therefore, we calculate the prices and profits only for the first two cases.

Case (i): Firm 1 sets the highest possible price, such that $\underline{x} = 1$. It follows $p_T^{d_1} = v - 2\tau$. In addition, it sets the fixed fee, such that the customer at location $x = 1$ is indifferent between buying and not buying, that is, $U_1\left(x = 1; p_1 = p_T^{d_1}\right) = 0$. Thus, it sets $f_T^{d_1} = \tau$. The profit is $\pi_T^{d_1} = v - \tau$.

Case (ii): Firm 1 maximizes its profit under the constraint $\overline{x} = 1$, which is equivalent to $f_T^{d_2} = (p_1 - v)^2/4\tau$. The maximization yields $p_T^{d_2} = v/3 + 2\tau/3$ and, hence, $f_T^{d_2} = (v-\tau)^2/9\tau$. The profit is $\pi_T^{d_2} = (-v^3 + 12v^2\tau + 6v\tau^2 + 10\tau^3)/54\tau^2$.

Next, we derive a threshold that determines which case would be optimal for firm 1, ignoring the existence of case (iii). The constraint

$$\underline{x}\left(p_1 = p_T^{d_2}, f_1 = f_T^{d_2}\right) \leq 1 \qquad \Leftrightarrow \qquad \tau \geq \frac{v}{4}$$

and the comparison of the profits

$$\pi_T^{d_1} < \pi_T^{d_2} \qquad \Leftrightarrow \qquad \tau > \frac{v}{4}$$

lead to the threshold.

Because we did not calculate prices and profits for the third case, the profits for the other two cases are lower bounds, that is, firm 1 might be able to achieve an even larger profit if it sets its price in a way such that some customers choose the outside option and do not buy. We establish an upper bound for the deviation profit in Lemma 4, where we investigate fully nonlinear tariffs. The reason is that with fully nonlinear tariffs, firm 1 would deviate from the same collusive prices (see Corollary 1), but has access to a tariff structure that is more flexible and nests two-part tariffs as a special case. Because the lower bound equals the upper bound for $\tau < v/4$, we can replace the inequality sign with an equality sign.

Proof (Lemma 3). The critical discount factors result immediately from inserting the respective profits into Expression (2). Competitive profits are derived by Anderson and Neven (1989) and Hoernig and Valletti (2007), whereas collusive and deviation profits are given by Lemma 1 and Lemma 2. Note that the critical discount factor (2) is monotonically increasing in the deviation profit, so that in the case of two-part tariffs, the lower bound for the profit results in a lower bound for the critical discount factor.

Proof (Proposition 1). The proposition results immediately from pairwise comparisons of the critical discount factors and the corresponding bounds.

Proof (Lemma 4). First, we introduce the notation in line with Hoernig and Valletti (2011, p. 6). Let

$$u(x,q) = v - v(1-q) - \tau(1-q-x)^2,$$
$$\tilde{U}(x,\hat{x}) = u(x,Q(\hat{x})) - P(\hat{x}), \text{ and}$$
$$U(x) = \tilde{U}(x,x).$$

The function $u(x,q)$ captures the residual utility when a customer at location x buys quantity q from firm 1 and quantity $1-q$ from firm 2 before paying the price of firm 1. It is useful to think about this residual utility as the largest possible amount that firm 1 can charge from this specific customer. $\tilde{U}(x,\hat{x})$ is the utility of a customer at location x when the customer buys the quantity that the customer at location $\hat{x}$ was supposed to buy. Finally, $U(x)$ is the utility of a customer at location x when the customer actually buys the supposed quantity.

The maximization problem is then given by:

$$\max_{P,Q}\left[\pi_1 = \int_0^{\bar{x}} P(x)\,dx\right]$$
$$\text{s.t. } (IC) \quad U(x) \geq \tilde{U}(x,\hat{x}) \quad \forall x,\hat{x} \in [0,\bar{x}],$$
$$(PC) \quad U(x) \geq 0 \quad \forall x \in [0,\bar{x}]$$

Note that our participation constraint differs from that in Hoernig and Valletti (2011). In their analysis, customers evaluate the possibility of buying from firm 1 against the possibility of buying from firm 2 exclusively. This is not reasonable in our analysis because firm 2 charges the collusive tariff $P(q) = vq$. If a customer buys exclusively from firm 2, the customer (at $x < 1$) would suffer from total costs of more than v, which exceeds the customer's willingness to pay. Therefore, customers who choose not to buy from firm 1 will not buy any product.

The beginning of our proof closely follows the first two steps of the proof of Proposition 1 in Hoernig and Valletti (2011). First note that the participation constraint binds for $x = \bar{x}$, that is,

$$u(\bar{x},Q(\bar{x})) - P(\bar{x}) = 0.$$

The incentive constraint has to bind for all customers $x \in [0,\bar{x}]$ and can be rewritten as follows

$$u(x,Q(x)) - P(x) \overset{def}{=} U(x) \overset{(IC)}{=} \tilde{U}(x,\bar{x}) \overset{def}{=} U(\bar{x}) - \int_x^{\bar{x}} \frac{\partial u(s,Q(s))}{\partial x} ds.$$

Note that the first and third equality follow from the definitions of the respective objects, while the second equality is just the binding incentive constraint. Rewriting this equality yields:

$$P(x) = u(x,Q(x)) + \int_x^{\bar{x}} \frac{\partial u(s,Q(s))}{\partial x} ds - U(\bar{x}).$$

The deviation profit of firm 1 is then given by integrating over all customers in $x \in [0, \bar{x}]$:

$$\pi_1 = \int_0^{\bar{x}} \left[u(x, Q(x)) + \int_x^{\bar{x}} \frac{\partial u}{\partial x}(s, Q(s))\, ds - U(\bar{x}) \right] dx$$
$$= \int_0^{\bar{x}} \left[u(x, Q(x)) + \int_x^{\bar{x}} \frac{\partial u}{\partial x}(s, Q(s))\, ds \right] dx - \bar{x} \cdot U(\bar{x}).$$

Hoernig and Valletti (2011) analytically show that one can further simplify this expression:

$$\pi_1 = \int_0^{\bar{x}} \left[u(x, Q(x)) + x \cdot \frac{\partial u}{\partial x}(x, Q(x)) \right] dx - \bar{x} \cdot U(\bar{x}). \tag{A.1}$$

This completes the first step of the proof in Hoernig and Valletti (2011). In their second step, they derive the first-order condition for the optimal $Q(x)$ depending on the rival's tariff. While the rival's tariff is unknown at this stage in their analysis, we know the optimal collusive tariff the rival will set. This leads to the following characterization of $Q(x)$:

$$v + 2\tau(1 - Q(x) - x) = 2\tau x \quad \Leftrightarrow \quad Q(x) = \frac{v}{2\tau} + 1 - 2x.$$

Note that we require $Q(x) \in [0, 1]$, which leads to the following thresholds:

$$Q(x) \geq 0 \quad \Leftrightarrow \quad x \leq \frac{v}{4\tau} + \frac{1}{2} \equiv \overline{\theta},$$
$$Q(x) \leq 1 \quad \Leftrightarrow \quad x \geq \frac{v}{4\tau} \equiv \underline{\theta}.$$

We can further check under which conditions the customers located at $\underline{\theta}$ and $\overline{\theta}$ are located in $[0, 1]$. First, note that $\underline{\theta} > 0$ and $\overline{\theta} > 0$ always hold. Second, we can express $\overline{\theta}$ in terms of $\underline{\theta}$, that is, $\overline{\theta} = 1/2 + \underline{\theta}$. Comparing $\underline{\theta}$ and $\overline{\theta}$ to one leads to two conditions for the product differentiation parameter:

$$\underline{\theta} \leq \frac{1}{2} \quad \Leftrightarrow \quad \tau \geq \frac{v}{2} \equiv \Theta_2$$
$$\underline{\theta} \leq 1 \quad \Leftrightarrow \quad \tau \geq \frac{v}{4} \equiv \Theta_1$$

We further have to ensure that $Q(x)$ is well-defined in the sense that a customer at x buying $Q(x)$ must have a (weakly) positive residual utility:

$$v - v \cdot (1 - Q(x)) - \tau(1 - Q(x) - x)^2 \geq 0 \quad \Leftrightarrow \quad x \in \left[\underbrace{-\frac{v\tau + \sqrt{4\tau^3 v + 2\tau^2 v^2}}{2\tau^2}}_{<0}, \underbrace{\frac{-v\tau + \sqrt{4\tau^3 v + 2\tau^2 v^2}}{2\tau^2}}_{\equiv \tilde{\theta}} \right].$$

It is also straightforward to verify that $\tilde{\theta} \leq \overline{\theta}$ always holds. Further constraints arise from the following comparisons:

$$\tilde{\theta} \geq \underline{\theta} \quad \Leftrightarrow \quad \tau \leq \frac{v}{16},$$
$$\tilde{\theta} \leq 1 \quad \Leftrightarrow \quad \tau \geq \frac{v}{2} = \Theta_2.$$

The constraints lead to four areas that we have to investigate in detail:

1. Area I ($\tau \leq v/16$): All customers buy exclusively from firm 1, that is, $1 \leq \tilde{\theta} \leq \underline{\theta} < \overline{\theta}$.
2. Area II ($v/16 < \tau \leq \Theta_1$): All customers buy exclusively from firm 1, that is, $1 \leq \underline{\theta} \leq \tilde{\theta} \leq \overline{\theta}$.
3. Area III ($\Theta_1 < \tau \leq \Theta_2$): Customers in $[0, \underline{\theta}]$ buy exclusively from firm 1, and customers in $[\underline{\theta}, 1]$ buy from both firms, that is, $\underline{\theta} \leq 1 \leq \tilde{\theta} \leq \overline{\theta}$.
4. Area IV ($\Theta_2 < \tau$): Customers in $[0, \underline{\theta}]$ buy exclusively from firm 1, and customers in $\left[\underline{\theta}, \tilde{\theta}\right]$ buy from both firms, that is, $\underline{\theta} \leq \tilde{\theta} \leq \overline{\theta} \leq 1$.

Note that the first two areas have the same outcome, so that we can pool them in one case.

Case 1 ($\tau \leq \Theta_1$): If the deviating firm monopolizes the market, it extracts the entire utility from the customer located at $\bar{x} = 1$, that is, $U(\bar{x}) = 0$.

$$P(\bar{x}) = u(\bar{x}, Q(\bar{x})) = v - \tau.$$

The resulting profit is

$$\pi_1 = \int_0^1 \left[\underbrace{v - \tau \cdot x^2}_{=u(x,Q(x))} + \underbrace{(-1) \cdot 2 \cdot \tau \cdot x^2}_{x \cdot \frac{\partial u}{\partial x}(x,Q(x))} \right] dx = \left[v \cdot x - \tau x^3\right]_0^1 = v - \tau.$$

Case 2 ($\Theta_1 \leq \tau \leq \Theta_2$): Customers in the interval $[0, \underline{\theta}]$ will buy exclusively from firm 1, whereas customers in $[\underline{\theta}, 1]$ will buy from both firms. This allows us to split the profit function of the deviating firm into two components. Further, note that $U(\bar{x}) = 0$ for $\bar{x} = 1$ because the deviating firm could otherwise gain a larger profit by charging a higher price:

$$\begin{aligned}
\pi_1 &= \int_0^1 \left[u(x, Q(x)) + x \cdot \frac{\partial u}{\partial x}(x, Q(x)) \right] dx \\
&= \int_0^{\underline{\theta}} \left[u(x, 1) + x \cdot \frac{\partial u}{\partial x}(x, 1) \right] dx + \int_{\underline{\theta}}^1 \left[u(x, Q(x)) + x \cdot \frac{\partial u}{\partial x}(x, Q(x)) \right] dx
\end{aligned}$$

We calculate the values of both components separately. For the first component, we get

$$\int_0^{\underline{\theta}} \left[\underbrace{v - \tau \cdot x^2}_{=u(x,Q(x))} + \underbrace{(-1) \cdot 2 \cdot \tau \cdot x^2}_{x \cdot \frac{\partial u}{\partial x}(x,Q(x))} \right] dx = \left[v \cdot x - \tau x^3\right]_0^{\underline{\theta}} = v \cdot \underline{\theta} - \tau \cdot \underline{\theta}^3 = \frac{v^2}{4\tau} - \frac{v^3}{64\tau^2}.$$

Turning to the second component, we get

$$\int_{\underline{\theta}}^1 \left[\underbrace{v\left(\frac{v}{2\tau} + 1 - 2x\right) - \tau\left(x - \frac{v}{2\tau}\right)^2}_{=u(x,Q(x))} + \underbrace{2\tau x\left(x - \frac{v}{2\tau}\right)}_{x \cdot \frac{\partial u}{\partial x}(x,Q(x))} \right] dx = \left[\frac{v^2 x}{4\tau} + vx - vx^2 + \frac{\tau x^3}{3}\right]_{\underline{\theta}}^1 = \frac{64\tau^3 - v^3}{192\tau^2}.$$

Adding up the values of both components, we get

$$\pi_1 = \frac{16\tau^3 + 12\tau v^2 - v^3}{48\tau^2}.$$

Case 3 ($\Theta_2 \leq \tau$): Finally, we turn to the case in which some customers choose the outside option and do not buy from any firm. By definition, we have $u(\bar{x}, Q(\bar{x})) = 0$ for $\bar{x} = \tilde{\theta}$. This implies $P(Q(\bar{x})) = 0$ and $U(\bar{x}) = 0$. As in the previous case, we can split the profit function into two components:

$$\begin{aligned}\pi_1 &= \int_0^{\tilde{\theta}} \left[u(x, Q(x)) + x \cdot \frac{\partial u}{\partial x}(x, Q(x))\right] dx \\ &= \int_0^{\underline{\theta}} \left[u(x, 1) + x \cdot \frac{\partial u}{\partial x}(x, 1)\right] dx + \int_{\underline{\theta}}^{\tilde{\theta}} \left[u(x, Q(x)) + x \cdot \frac{\partial u}{\partial x}(x, Q(x))\right] dx.\end{aligned}$$

For the first component, we get

$$\int_0^{\underline{\theta}} \left[\underbrace{v - \tau \cdot x^2}_{=u(x,Q(x))} + \underbrace{(-1) \cdot 2 \cdot \tau \cdot x^2}_{x \cdot \frac{\partial u}{\partial x}(x,Q(x))}\right] dx = \left[v \cdot x - \tau x^3\right]_0^{\underline{\theta}} = \frac{v^2}{4\tau} - \frac{v^3}{64\tau^2}.$$

Turning to the second component, we get

$$\begin{aligned}\int_{\underline{\theta}}^{\tilde{\theta}} \left[\underbrace{v\left(\frac{v}{2\tau} + 1 - 2x\right) - \tau\left(x - \frac{v}{2\tau}\right)^2}_{=u(x,Q(x))} + \underbrace{2\tau x\left(x - \frac{v}{2\tau}\right)}_{x \cdot \frac{\partial u}{\partial x}(x,Q(x))}\right] dx &= \left[\frac{v^2 x}{4\tau} + vx - vx^2 + \frac{\tau x^3}{3}\right]_{\underline{\theta}}^{\tilde{\theta}} \\ &= \frac{5\left(\frac{\sqrt{\tau^2 v(2\tau+v)}\,(5v+4\tau)\sqrt{2}}{5} - \frac{45v\tau\left(v+\frac{48\tau}{25}\right)}{32}\right)v}{6\tau^3}.\end{aligned}$$

Adding up the values of both components, we get

$$\pi_1 = \frac{5\left(\frac{\sqrt{\tau^2 v(2\tau+v)}\,(5v+4\tau)\sqrt{2}}{5} - \frac{57v\tau\left(v+\frac{32\tau}{19}\right)}{40}\right)v}{6\tau^3}.$$

Critical discount factors: The critical discount factor results immediately from inserting the different types of profit into Expression (2).

Proof (Proposition 2). Note that Proposition 1 shows that the critical discount factor is always larger under two-part tariffs than under linear prices or fixed fees. Thus, the proposition immediately results from the comparison of $\overline{\delta}_T$ and $\overline{\delta}_N$.

A comparison of the competitive profits reveals that competitive profits are larger under fully nonlinear prices than under two-part tariffs. In addition, collusive prices and profits are the same and deviation profits are larger under fully nonlinear prices. The latter follows from the fact that the collusive prices are the same and that fully nonlinear tariffs nest two-part tariffs as a special case. Since the critical discount factor (2) increases in both the competitive and deviation profits, it follows $\overline{\delta}_N > \overline{\delta}_T$.

Proof (Corollary 2). Our analysis is similar to that in Chang (1991), except that customers in our model have the possibility to combine products. In the following, we show that this possibility does not change the result of Chang (1991).

In the first step, we assume that firms set collusive prices above the competitive prices (as in Chang, 1991). This means that all customers buy exclusively from one firm and mixing does not occur. To adopt the results of Chang (1991), we have to show that a deviating firm has no incentive to set prices that induce customers to mix. To see this, assume that the price is such that at least some customers combine products of both firms, that is,

$$\underline{x} < \overline{x} \quad \Leftrightarrow \quad \sqrt{\frac{f_2}{\tau}} < 1 - \sqrt{\frac{f_1}{\tau}} \quad \Leftrightarrow \quad \sqrt{\frac{f_1}{\tau}} < 1 - \sqrt{\frac{f_2}{\tau}}.$$

Consider the case in which firm 2 sets the collusive price $f_2 \geq f_F^* = \tau$ and firm 1 deviates. Then, $\sqrt{f_2/\tau} \geq 1$ and the right-hand side is not positive. Because we are only interested in the positive values of the square roots, the inequality cannot be satisfied irrespective of f_1. This implies that the deviating firm sets its price, such that all customers buy exclusively from one firm.

Second, we have to show that colluding firms have no incentive to set prices below the competitive prices to induce customer to mix. We have to distinguish between two cases. In the first case, firms set prices below competitive prices, but the prices are so large that customer do not mix. In this case, they would earn a profit that is smaller than the competitive profit (that is, $f_F^c/2$ instead of $f_F^*/2$), and because the competitive prices constitute a Nash equilibrium, they could always set these prices without the risk that the other firm could profitably deviate.

The second case covers prices below competitive prices that induce (at least some) customers to combine products from both firms. A necessary conditions for this customer behavior is $C \equiv \overline{x} - \underline{x} \geq 0$. The joint profit function is given by $\pi = f_1 \cdot \overline{x} + f_2 \cdot \underline{x}$. The derivative of the profit with respect to each fixed fee is independent of the other fixed fee and is strictly positive as long as $f_1 < 4\tau/9$ and $f_2 < 4\tau/9$. Because $f_1 = 4\tau/9$ and $f_2 = 4\tau/9$ violates the condition $C \geq 0$, we face a corner solution, where the condition on C binds with equality (that is, $C = 0$). We can rewrite C and express f_1 in terms of f_2, insert $f1\,(f_2)$ into the profit π, and use the first-order conditions on π with respect to f_2 to determine the optimal fixed fees. The result is $f_1 = f_2 = \tau/4$. This leads to an upper bound for the profit in the case of mixing customers of $\tau/8$. Comparing this upper bound to the collusive profits derived by Chang (1991) reveals that firms prefer to collude on prices above the competitive prices.

Proof (Lemma 5). Consider the case in which the firms want to sustain collusion for a given discount factor δ. They have to set a collusive price p_L^c, such that collusion is sustainable. We start with the analysis of the optimal behavior of a deviating firm. Similar to the case of collusion on profit-maximizing prices, there are two possible scenarios. First, the deviating firm sets a price, such that it captures the entire market.

Then, the deviating firm would set a price of $p_L^c - 2\tau$, which follows from the fact that the monopolization of the market requires $\underline{x} \leq 1$ and that the deviating firm wants to set the highest possible price given this constraint such that $\underline{x} = 1$. The resulting profit would be $p_L^c - 2\tau$. Alternatively, the deviating firm could leave the other firm with a strictly positive market share ($\underline{x} < 1$). The first-order condition implies that the price would be

$$p_L^{d,p} = \frac{2p_L^c - 4\tau + C}{3},$$

and the profit would be

$$\pi_L^{d,p} = \frac{(2p_L^c - 4\tau + C) \cdot \left(2\tau C - p_L^c C - 20\tau^2 - 4\tau p_L^c + (p_L^c)^2\right)}{108\tau^2},$$

with $C := \sqrt{28\tau^2 - 4\tau p_L^c + (p_L^c)^2}$. A comparison of the profits in both cases reveals that the profit in the second case is always strictly larger than the profit in the first case.

For a given price level p_L^c, we can calculate a critical discount factor using Expression (2). The optimal collusive price is then given by

$$p_L^c(\delta) = \arg\max_{p_L^c} \pi_L^{c,p} \quad \text{s.t.} \quad \delta \geq \frac{\pi_L^{d,p} - \pi_L^{c,p}}{\pi_L^{d,p} - \pi_L^*},$$

where the collusive profit is given by $\pi_L^{c,p} = p_L^c/2$ and the competitive profit is the same as before.

The expression on the right-hand side of the constraint is strictly decreasing in the price p_L^c for $p_L^c > \tau$. To see this, we can take the derivative of the expression with respect to the collusive price and then compare the derivative to zero. This comparison reveals that the derivative is always negative for $p_L^c > \tau$. Note that τ is the competitive price level and that firms would never charge a price below this level because they would prefer to compete otherwise. Because the collusive profit is strictly increasing in p_L^c, this implies that the colluding firms will choose the largest possible price, such that the constraint binds with equality, that is, p_L^c is defined by

$$\delta = \frac{\pi_L^{d,p} - \pi_L^{c,p}}{\pi_L^{d,p} - \pi_L^*}.$$

Because of the strict monotonicity, p_L^c is also uniquely defined by this equality.

Finally, it remains to show that the optimal collusive price $p_L^c(\delta)$ is decreasing in the discount factor. As noted above, the right-hand side of the above equation is strictly decreasing in the price. We can think of δ as a function of p_L^c. Because of the strict monotonicity, this function is bijective and its inverse function exists, with the inverse being also strictly decreasing. This concludes the proof.

Proof (Proposition 3). If firms collude on profit-maximizing prices under both pricing regimes (that is, if $\delta > \overline{\delta}_F$ and $\delta > \overline{\delta}_L$), profits are always larger with linear prices ($= v/2$) than with fixed prices

$(= v/2 - \tau/8)$. This also means that if collusion on profit-maximizing prices is sustainable with linear prices, but not with fixed prices (that is if $\delta < \overline{\delta}_F$ and $\delta > \overline{\delta}_L$), profits are larger with linear prices $(= v/2)$ than with fixed prices $(< v/2 - \tau/8)$.

Next, we look at the case in which collusion on profit-maximizing prices is sustainable with fixed prices, but not with linear prices (that is, if $\delta > \overline{\delta}_F$ and $\delta < \overline{\delta}_L$). With fixed fees, firms will set a price of $v - \tau/4$ and earn a profit of $v/2 - \tau/8$. Let us assume that with linear prices, firms would set the linear price to the same price level, that is, $p = v/2 - \tau/8$. Then, they would earn the same profit. It is straightforward to show that the critical discount factor that results from inserting this price in the formula stated in Lemma 5 is lower than $\overline{\delta}_F$. Because the critical discount factor stated in Lemma 5 is monotonically decreasing in the price, it follows that firms can increase the linear price until the critical discount factor equals the actual discount factor. This means that the profits will be larger with linear prices than with fixed prices.

Finally, we look at the case in which full collusion is never possible (that is, if $\delta < \overline{\delta}_F$ and $\delta < \overline{\delta}_L$). Let $f_F^c(\delta)$ and $p_L^c(\delta)$ be the optimal collusive prices that allow the firms to sustain collusion with fixed fees and linear prices for a given discount factor of δ. Then, each firm earns $f_F^c(\delta)/2$ with fixed fees and $p_L^c(\delta)/2$ with linear prices. This is because with fixed prices, customers do not mix and thus each firm covers half of the market. By contrast, with linear prices, all customers combine the products optimally, which also leads to a market share of 1/2. Because both profits result from dividing the corresponding prices by 2, it is sufficient to compare the levels of the optimal prices instead of the profits.

Lemma 5 does not provide a closed-form solution for $p_L^c(\delta)$, so that we cannot compare the resulting prices for a given discount factor. Instead, we will show that for a given price level p, the discount factor is lower with linear prices than with fixed fees; that is, if we rewrite $p = f_F^c(\delta)$ and $p = p_L^c(\delta)$ for δ, δ is larger in the first case with fixed fees than in the second case with linear prices. Because of the strictly monotonic relationship between the discount factors and the prices, this means that for a given discount factor δ, the price level is larger with linear prices than with fixed fees.

Lemma 5 provides the discount factor for the case of linear prices (set p_L^c to p). We can rewrite the optimal fixed fee from Corollary 2 and get

$$\delta = \begin{cases} \frac{f-2\tau}{2f-3\tau} & \text{if } \delta < \frac{1}{3} \\ \frac{f-\tau}{f+3\tau} & \text{if } \delta \geq \frac{1}{3}. \end{cases}$$

The comparison of the discount factors is straightforward and leads to the proposition.

Appendix B. Numerical simulation

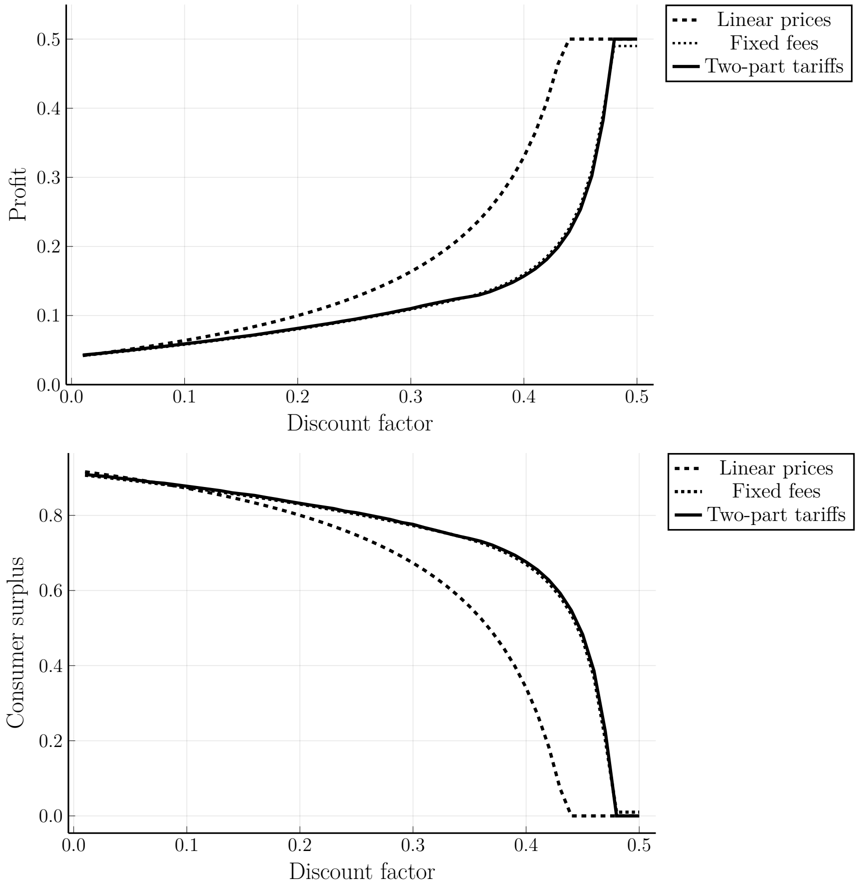

Figure B.3: Profits and customer surpluses under partial and full collusion ($v = 1$, $t = 0.08$).

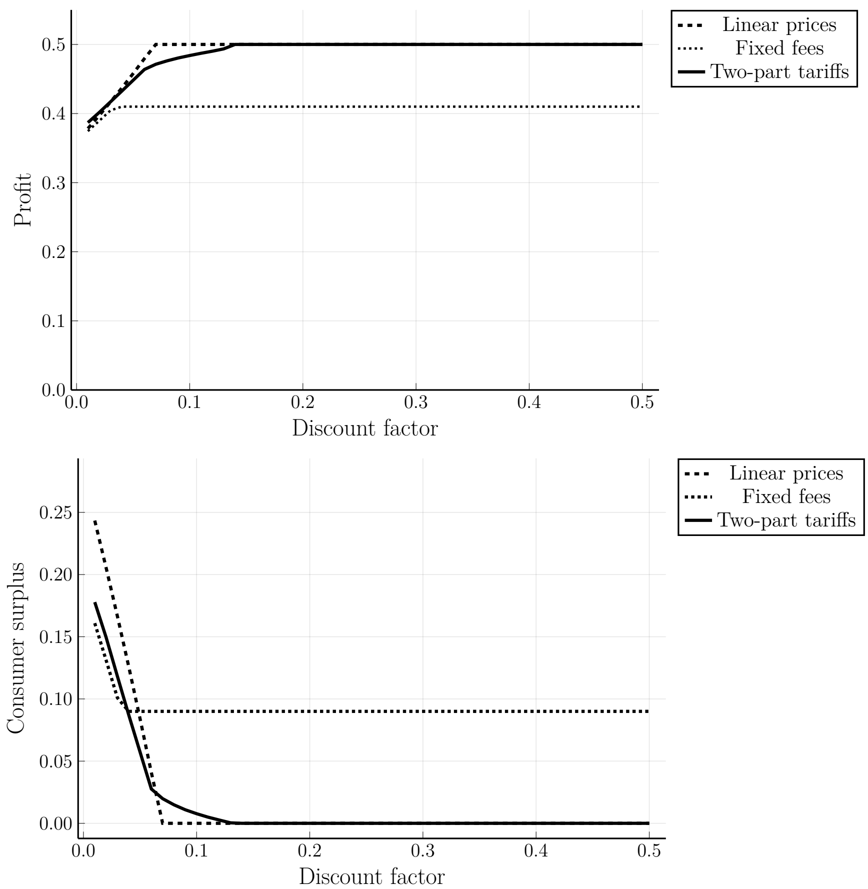

Figure B.4: Profits and customer surpluses under partial and full collusion ($v = 1$, $t = 0.72$).